Automatic poofs of Ramanujan series#

(using modular polynomals)

```
> restart:
  gc( ):

> RED:=g->cat(`#mn(\"`,g,`",mathcolor=\"#FF0000\")`):
  BLACK:=g->cat(`#mn(\"`,g,`",mathcolor=\"#0C090a\")`):
  GREEN:=g->cat(`#mn(\"`,g,`",mathcolor=\"#227d3f\")`):

  #
  fullsimplify := proc(expression) combine(evalc(simplify(expand(combine(rationalize(radnormal
  (expand(
  simplify(rationalize(simplify(combine(radnormal(expand(expression)), radicals)))))), radicals)))
  ), radicals);
  end:
> aboutthis:=proc()
> print(RED(`AUTOMATIC PROOFS OF RAMANUJAN-TYPE SERIES`)); print(RED(`(using modular Polynomials)`)
  );
> print(BLACK(`Jesús Guillera`));
> print(BLACK(`University of Zaragoza`));
> print(BLACK(`Written in 2023-24`));
  print(BLACK(`REFERENCE:`)-GREEN(`The fastest series for 1/pi due to Ramanujan. Proofs by modular
  polynomials`)); print();
> end:

> acknowledgements:=proc()
> print(RED(`MANY THANKS TO...`));
> print(BLACK(`ALIN BOSTAN`)-GREEN(`for his great idea of simplifying using an algebraic rule`));
> print(BLACK(`DREW SUTHERLAND`)-GREEN(`who answers my question about which coefficients of the
  Weber polynomials can be nonzero`));
> print(BLACK(`DORON ZEILBERGER`)-GREEN(`who invited me giving a ZOOM talk about this program April
  27 2023`));
  print(BLACK(`WADIM ZUDILIN`)-GREEN(`for sharing with me a generalization of the Legendre's
  relation`));
> print(BLACK(`JORGE ZÚÑIGA`)-GREEN(`for checking this program with many examples and detecting an
  error that had ocurred very few times`)); print();
> end:

> tryproc:=proc()
> print(RED(`USE OF MAIN PROCEDURE`));
> print(BLACK(`RamaPi(level=1,2,3,4, odd prime, degree roots=1,2,3,4, RUSSELL or WEBER)`));
> print(BLACK(`RUSSELL for levels 2,3,4; WEBER for levels 1,2,4`));
  print();
> end:
>
> aboutthis();
```

AUTOMATIC PROOFS OF RAMANUJAN-TYPE SERIES

(using modular Polynomials)

Jesús Guillera

University of Zaragoza

Written in 2023-24

REFERENCE: − The fastest series for 1/pi due to Ramanujan. Proofs by modular polynomials

(1)

```
> acknowledgements();
```


MANY THANKS TO...

ALIN BOSTAN − for his great idea of simplifying using an algebraic rule

DREW SUTHERLAND − who answers my question about which coefficients of the Weber polynomials can be nonzero

DORON ZEILBERGER − who invited me giving a ZOOM talk about this program April 27 2023

WADIM ZUDILIN − for sharing with me a generalization of the Legendre's relation

JORGE ZÚÑIGA − for checking this program with many examples and detecting an error that had ocurred very few times

(2)


```
> tryproc();
```

USE OF MAIN PROCEDURE

RamaPi(level=1,2,3,4, odd prime, degree roots=1,2,3,4, RUSSELL or WEBER)

RUSSELL for levels 2,3,4; WEBER for levels 1,2,4

(3)

Legendre's relation:

```
> legendre:= proc(lev,alpha)
  local leve,beta,F,G,leg,s;
```

```
if lev = 1 then s := 6; end if;
if lev = 2 then s := 4; end if;
if lev = 3 then s := 3; end if;
if lev = 4 then s := 2; end if;
leve := 4*sin(Pi/s)^2;
F := xx -> subs(x = xx, hypergeom([1/s, 1 - 1/s], [1], x));
G := xx -> subs(x = xx, x*diff(F(x), x));
beta := 1 - alpha;
leg := 2*alpha*F(alpha)*G(beta)/sqrt(leve) + 2*beta*F(beta)*G(alpha)/sqrt(leve);
print();
print(simplify(leg, hypergeom) = 1/Pi);
print();
end:
```

Examples

```
> legendre(2,-1/5);
```

$$-\frac{1}{200}\left(9\sqrt{2}\left(\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],[1],-\frac{1}{5}\right)\text{hypergeom}\left(\left[\frac{5}{4},\frac{7}{4}\right],[2],\frac{6}{5}\right)+\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],[1],\frac{6}{5}\right)\text{hypergeom}\left(\left[\frac{5}{4},\frac{7}{4}\right],[2],-\frac{1}{5}\right)\right)\right)=\frac{1}{\pi} \quad (4)$$

```
> legendre(3,5/3);
```

$$-\frac{1}{243}\left(40\sqrt{3}\left(\text{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],[1],\frac{5}{3}\right)\text{hypergeom}\left(\left[\frac{4}{3},\frac{5}{3}\right],[2],-\frac{2}{3}\right)+\text{hypergeom}\left(\left[\frac{1}{3},\frac{2}{3}\right],[1],-\frac{2}{3}\right)\text{hypergeom}\left(\left[\frac{4}{3},\frac{5}{3}\right],[2],\frac{5}{3}\right)\right)\right)=\frac{1}{\pi} \quad (5)$$

Clausen's identity

```
> clausen:=proc(lev,x)
local R,F,clau,s:
s:=Pi/arcsin(sqrt(lev)/2):
R := x -> hypergeom([1/2, 1/s, 1 - 1/s], [1, 1], 4*x*(1 - x));
F := x -> hypergeom([1/s, 1 - 1/s], [1], x);
clau := x -> F(x)^2 - R(x);
print(); print(simplify(clau(x), hypergeom) = 0);
end:
```

Examples

```
> clausen(2,1/4);
```

$$\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],[1],\frac{1}{4}\right)^2-\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4}\right],[1,1],\frac{3}{4}\right)=0 \quad (6)$$

```
> clausen(2,-2);
```

$$\text{hypergeom}\left(\left[\frac{1}{4},\frac{3}{4}\right],[1],-2\right)^2-\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4}\right],[1,1],-24\right)=0 \quad (7)$$

Russell modular equations corresponding to $\ell\in\{2,3,4\}$

```
> russellmodequ:=proc(lev,p)
local uu,vv,mm,x,s,val,qq,m,coe,k,i,num,aalpha,bbeta,alpha,beta,sb,u,v,suma,ecus,ecucoef,formu,
sol,o,q:
global P,hh,ro,BC,h,dpol,dec,pol,T,llev,dd:
dec:=p:
o:=time():
if lev = 2 then
    s := 4;
    ro := 256;
    BC := n -> (4*n)!/(n!)^4;
    val := simplify((p + 1)/4);
```

```
    h := 4/denom(val);
    dpol := 2*val;
end if;
if lev = 2 and dec = 3 then
    h := 1;
    dpol := 4;
end if;
if lev = 3 then
    s := 3;
    ro := 108;
    BC := n -> (3*n)!*(2*n)!/(n!)^5;
    val := simplify((p + 1)/3);
    h := 6/denom(val);
    dpol := numer(val);
end if;
if lev = 3 and p=2 then h:=3; dpol:=1; end if;
if lev = 3 and p=7 then h:=2; dpol:=8; end if:
if lev = 4 then
    s := 2;
    ro := 64;
    BC := n -> ((2*n)!)^3/(n!)^6;
    val := simplify((p + 1)/8);
    h := 8/denom(val);
    dpol := numer(val);
end if;
if lev = 4 and d = 3 then
    h := 1;
    dpol := 4;
end if;
if lev = 4 and d = 1 then
    h := 1;
    dpol := 2;
end if;
coe := seq(seq(c[k - i, i], i = 0 .. k), k = 1 .. dpol);
num := nops({coe});
qq := 4*exp(int(1/(hypergeom([1/2, 1/s, 1 - 1/s], [1, 1], 4*x*(1 - x))*x*(1 - x)), x))/ro;
hh := h;
Order := 2*num;
assume(q, real); assume(q,positive); mm := convert(solve(series(qq, x) = q, x), polynom);
print(); print(GREEN(LEVEL)= lev, GREEN(`     DEGREE`) = p);
print();
print(GREEN(`MODULAR EQUATION`)); print();
aalpha := subs(q = q^p, mm); bbeta := mm;
uu := convert(series(aalpha^(1/h)*bbeta^(1/h), q, 2*num), polynom);
vv := convert(series((1 - aalpha)^(1/h)*(1 - bbeta)^(1/h), q, 2*num), polynom);
suma := sum(sum(c[k - i, i]*uu^i*vv^(k - i), i = 0 .. k), k = 1 .. dpol);
ecus := series(suma, q, 2*num) - 1;
ecucoef := coeffs(simplify(convert(ecus, polynom)), q);
formu := sum(sum(c[k - i, i]*u^i*v^(k - i), i = 0 .. k), k = 1 .. dpol) - 1;
sol := solve({ecucoef}, {coe});
pol := sort(subs(sol, formu), [u, v]);
P := (uu, vv) -> subs(u = uu, v = vv, pol);
dec := p; dd := p; llev := lev;
print(u^h = alpha*beta,cat(`    `,v)^h = (1 - alpha)*(1 - beta),`   P(u,v)=0,`);
print(`P(u,v)`= pol); print();
print(GREEN(SECONDS = time() - o));
print(GREEN(`______________________________________________________________`));
print();
end:
```

Weber modular equation

```
> with(IntegerRelations):
webermodequ:=proc(lev,p)
local o,i,j,nume1,nume2,sol,formu,fw,utau0,vtau0,eta,se,sse,ss,LL,T,S,W,pp,pol:
global P:
o:=time():
print(); print(GREEN(LEVEL)= lev, GREEN(`     DEGREE`) = p);
print(); print(GREEN(`MODULAR EQUATION`));
print();
eta := tau -> exp(1/12*I*Pi*tau)*Product(1 - exp(2*I*Pi*tau*n), n = 1 .. infinity);
fw := eta(tau)^2/(eta(tau/2)*eta(2*tau)); Digits := (p + 1)^2/2;
utau0 := evalf(subs(tau = exp(-1)*I, fw));
vtau0 := evalf(subs(tau = exp(-1)*p*I, fw));
LL := [];T := [];
for i from 0 to p + 1 do
   for j from 0 to p + 1 do
           if ((i*p + j) + (-p - 1)) mod 24 <> 0 then NULL else
                LL := [op(LL), utau0^i*vtau0^j];
                T := [op(T), u^i*v^j];
           end if;
    end do;
end do;
nume1 := nops(LL);nume2 := nops(T);
S := PSLQ(LL);formu := sum(S[k]*T[k], k = 1 .. nume1);
assume(uu, positive);assume(vv, positive);
```

```
W := solve(subs(sqrt(uu*vv) = t, combine(subs({u = sqrt(uu), v = sqrt(vv)}, formu), radicals)),
t)^2;
pol := uu*vv*denom(W) - numer(W);
P := (u, v) -> subs({uu = u, vv = v}, pol);
if lev=1 then
   print(alpha*(1 - alpha) = 432*u^12/(u^12 - 16)^3, beta*(1 - beta) = 432*v^12/(v^12 - 16)^3,`
P(u,v)=0,`);
end if:
if lev=2 then
   print(alpha*(1 - alpha) = -256*u^12/(64-u^12)^2, beta*(1 - beta) = -256*v^12/(64-v^12)^2,`  P
(u,v)=0,`);
end if:
if lev=4 then
   print(alpha*(1 - alpha) = 16/u^12,beta*(1 - beta)=16/v^12, `P(u,v)=0,`);
end if:
print(); print(`P(u,v)`  =P(u,v)): print();
print(GREEN(SECONDS = time() - o));
print(GREEN(`________________________________________________________________`));
print();
end:
>
```

Modular equations in Russell ℓ∈{2 , 3 , 4} or Weber ℓ ∈{1 , 2, 4}, p ∈{odd primes)

```
> russellmodequ(2,11);
```

LEVEL = 2, DEGREE = 11

MODULAR EQUATION

$$u^4=\alpha\,\beta,\quad v^4=(1-\alpha)\,(1-\beta),\quad P(u,v)=0,$$

$$P(u,v)=u^6+1996\,u^5\,v-3021\,u^4\,v^2+9176\,u^3\,v^3-3021\,u^2\,v^4+1996\,u\,v^5+v^6-3\,u^4+1896\,u^3\,v-6198\,u^2\,v^2+1896\,u\,v^3-3\,v^4+3\,u^2+204\,u\,v+3\,v^2-1$$

SECONDS = 3.250

---

(8)

```
> webermodequ(2,11);
```

LEVEL = 2, DEGREE = 11

MODULAR EQUATION

$$\alpha\,(1-\alpha)=-\frac{256\,u^{12}}{(-u^{12}+64)^2},\ \beta\,(1-\beta)=-\frac{256\,v^{12}}{(-v^{12}+64)^2},\quad P(u,v)=0,$$

$$P(u,v)=u\,v\,(u^5\,v^5-11\,u^4\,v^4+44\,u^3\,v^3-88\,u^2\,v^2+88\,u\,v-32)^2-(u^6+v^6)^2$$

SECONDS = .141

---

(9)

This computes $\frac{dv}{du}$, and $\frac{d^2v}{du^2}$, and evaluates them at u_0

```
> dvddv:=proc(dd)
  #
  local sols,Sols,TT,dY,dP,nn,nunu,dede,NUNU,DEDE,data1,data2,lim,ddP,d2P,LA,LE;
  global Dv,DDv,T,dv,ddv,dv0,ddv0,ddvu,atu0,r0,Dv0,Ddv0,ecu,CONTA;
  T := u -> P(u, v(u));
  atu0 := {u = u0, diff(v(u), u) = dv0, diff(v(u), u, u) = ddv0, v(u) = v0};
  Ddv0:=simplify(evala(subs({u=u0,v(u)=v0},simplify(Ddv,rulepol))),rulepol);
  if Ddv0<>0 then
  dv := simplify(factor(solve(diff(T(u), u), diff(v(u), u))));
  ddv := simplify(factor(subs(diff(v(u),u)=dv,diff(dv, u))));
  dv0:=simplify(evala(subs(atu0,dv)),rulepol);
  ddv0:=simplify(evala(subs(atu0,ddv)),rulepol);
  else
> dP:=solve(diff(P(u,v(u)),u),diff(v(u),u)):
> nunu:=numer(dP): dede:=denom(dP):
> eval(subs(u=u0,v=v0,nunu)); eval(subs(u=u0,v=v0,dede));
```

```
> sols:=fullsimplify(solve(simplify(subs(u=u0,subs(v(u)=v0,subs(diff(v(u),u)=LA,diff(nunu,u)/diff
  (dede,u)))))-LA,LA));
  if nops([sols])=1 then dv0:=simplify(sols,rulepol) else dv0:=simplify(sols[1],rulepol); end if;
> data1:=u=u0,v(u0)=v0,D(v)(u0)=dv0:
> ddP:=diff(P(u,v(u)),u,u):
> ddP:=simplify(solve(ddP,diff(v(u),u,u))):
> d2P:=convert(ddP,D):
> nunu:=simplify(numer(d2P)): dede:=simplify(denom(d2P)):
> lim:=simplify(limit(eval(factor(diff(nunu,u)/diff(dede,u))),u=u0)):
> ecu:=simplify(subs((D@@2)(v)(u0)=LU,simplify(subs(data1,lim))))=LU;
> Sols:=simplify(evala(fullsimplify(solve(ecu,LU))));
  if nops([Sols])=1 then ddv0:=simplify(Sols,rulepol) else ddv0:=simplify(Sols[1],rulepol); end if;
  end if;
> end:
```

\# Calculating α , β , $\frac{d\alpha}{du}$, $\frac{d\beta}{du}$, $\frac{d^2\alpha}{du^2}$, $\frac{d^2\beta}{du^2}$ at u_0 \#

\> # For Russell polynomials #

```
>
> alpha0beta0russell:=proc(lev,dd)
  global T,alpha0,beta0,z0,dalpha0,dbeta0,ddalpha0,ddbeta0,m0,Ddv0:
  alpha0 := solve(alpha*(1 - alpha) = expand(u0^h), alpha)[1];
  beta0 := 1 - alpha0;
  z0 := fullsimplify(4*alpha0*beta0);
  #
  dalpha0 := evalc(fullsimplify(expand(subs(u=u0,subs(beta(u)=beta0,subs(alpha(u)=alpha0,subs(v(u)=
  v0,subs(diff(v(u),u)=dv0,
  simplify(expand((alpha(u)*(h*u^(h-1)-h*v(u)^(h-1)*diff(v(u),u))-h*u^(h-1))/(alpha(u)-beta(u))))))
  ))))));
  #
  dbeta0 := evalc(fullsimplify(expand(h*u0^(h-1)-h*v0^(h-1)*dv0-dalpha0)));
  ddalpha0 := fullsimplify(expand(solve(h*(h-1)*u0^(h-2)-2*dalpha0*dbeta0-x*beta0-alpha0*(h*(h-1)*
  u0^(h-2)-h*(h-1)*v0^(h-2)*dv0^2-h*v0^(h-1)*ddv0-x),x)));
  ddbeta0 := fullsimplify(expand(h*(h - 1)*u0^(h - 2) - h*(h - 1)*v0^(h - 2)*dv0^2 - h*v0^(h - 1)*
  ddv0 - ddalpha0));
  end:
>
```

\> # For Weber polynomials #

```
>
  alpha0beta0weber:=proc(lev,dd)
  local q: global u0,v0,alpha0,beta0,dalpha0,dbeta0,ddalpha0,ddbeta0,m0,dm0,atu0,tau0,q0,Ddv0:
  alpha0 := solve(alfa*(1 - alfa) = simplify(w0), alfa)[1];
  beta0 := 1 - alpha0;
  dalpha0 := simplify(evala(dw0/(1 - 2*alpha0)));
  dbeta0 := simplify(evala(dy0/(1 - 2*beta0)));
  ddalpha0:=simplify(evala((2*dalpha0^2 + ddw0)/(1 - 2*alpha0)));
  ddbeta0 := simplify(evala((2*dbeta0^2 + ddy0)/(1 - 2*beta0)));
  end:
```

\# Computing tau, sequences of degrees and the parameters b and a \#

```
> with(polytools):
  mba:=proc(lev,dd,polclass)
  local o,BC,FC,SDIV,SD,BB,R,RH,q0,ii,posd;
  #
  global u0,v0,T,w0,dv0,alpha0,beta0,z,dalpha0,dbeta0,m0,ddv0,ddalpha0,rootone,
  dy,ddy,a1,a2,ro,dw0,dy0,ddw0,ddy0,c0,ddbeta0,b0,a0,J0,atu0,w,y,y0,dv,ddv,dw,ddw,
  zp,z0,dm0,diver,tau0,imtau0,SDIVER,dec,VRH,GFACTOR,FG,b1,vz0,tauR0,COMP,prideg,
  IFACT,CC,CR,del,n0,r0,g0,pr,DELTA,DELTA2,Ddv0,M0,D0,CONTA:
  #
  o := time(); dec:=dd:
  #
  if polclass=WEBER then
    if lev=1 then
      ro:=1728: BC := n -> (6*n)!/((3*n)!*(n!)^3);
      w:= 432*u^12/(u^12-16)^3; y:=432*v^12/(v^12-16)^3;
    end if:
    if lev=2 then
      ro:=256: BC := n -> (4*n)!/(n!)^4;
      w := -64*u^12/(64-u^12)^2; y:=-64* v^12/(64-v^12)^2;
    end if:
    if lev=4 then
      ro:=64: BC := n -> (2*n)!^3/(n!)^6;
      w :=u^12/256;y :=v^12/256;
    end if:
    dvddv(dd);
    dw := diff(w, u);  ddw := diff(dw, u);
    dy := diff(y, v);
```

```
    ddy :=diff(dy, v); w0 := subs(u=u0,w);
    y0 := subs(v=v0,y);
    dw0:=subs(u=u0,simplify(dw,rulepoli));
    ddw0:=subs(u=u0,simplify(ddw,rulepoli));
    dy0:=fullsimplify(subs(v=v0,simplify(dy,rulepoli))*dv0);
    ddy0:=fullsimplify(subs(v=v0,simplify(ddy,rulepoli))*dv0^2+subs(v=v0,dy)*ddv0);
    T := u -> P(u, v(u));
    alpha0beta0weber(lev, dd);
  end if;
  #
  if polclass=RUSSELL then
    if lev=3 then ro:=108: BC := n -> (3*n)!*(2*n)!/n!^5; end if:
    if lev=2 then ro:=256: BC := n -> (4*n)!/(n!)^4; end if:
    if lev=4 then ro:=64:  BC := n -> (2*n)!^3/(n!)^6; end if:
    atu0 := {u = u0, diff(v(u), u) = dv0, diff(v(u), u, u) = ddv0, v(u) = v0};
    dvddv(dd);
    T := u -> P(u, v(u)); alpha0beta0russell(lev, dd);
  end if;
  Ddv0:=simplify(evala(subs({u=u0,v(u)=v0},simplify(Ddv,rulepol))),rulepol);
  #
  if polcl=WEBER or polcl=RUSSELL then
  #
    m0 := combine(expand(convert(simplify(convert(fullsimplify(sqrt(fullsimplify(dalpha0/(dd*
  dbeta0)))),RootOf)),radical)),radicals);
    tau0:=simplify(evala(evalc(combine(simplify(evala(I*m0*dec/sqrt(lev))),radicals))));
    J0 := simplify(evala(ro/z0));
    b0:=simplify(evala((1-2*alpha0)*(m0*dec+1/m0)*1/sqrt(lev)));
    c0:=simplify(evala(rationalize(dec/(b0*sqrt(lev))*m0*alpha0*beta0/dalpha0*(ddbeta0/dbeta0-
  ddalpha0/dalpha0))));
    a0:=simplify(evala(b0*(1+c0)));
    DELTA:=simplify(bb0/b0); DELTA:=simplify(evala(Re(M0)/Re(m0)*D0/d0));
    assume(abs(ro/J) < 1);
    BB:=convert(Sum(BC(n)*(b0*n + a0)/J^n, n = 0 .. infinity), hypergeometric);
    R:=Sum(BC(n)*(simplify(DELTA*b0)*n + simplify(DELTA*a0))/J0^n,n=0..infinity);
    RH:= subs(J = J0, BB); Digits:=100;
    q0:=simplify(evala(exp(2*Pi*I*tau0))); g0:=lev*abs(tau0^2);
  #
    if type(DELTA^2,numeric) then
      print(CONTA,nops(M)); print(GREEN(`SPECIAL VALUE of z`)=z0);
      tau0:=simplify(evala(I/abs(1/DELTA)*dec*m0/(sqrt(lev))));
      print(GREEN(`multiplier`) = m0,GREEN(`       tau`)=tau0,GREEN(`   degree`)=1/abs(m0)^2); print
  (delta=DELTA);
      if Re(tau0)=0 then pr:=tau0 fi;
      if Re(tau0)>0 then pr:=tau0-floor(Re(tau0)); fi;
      if Re(tau0)<=0 then pr:=tau0+floor(-Re(tau0)); fi;
      prideg:=simplify(lev*(abs(pr)^2));
      print(GREEN(`sequence of possible degrees for n=0,1,2,... corresponding to the same`)+delta);
      print(simplify((1/DELTA)^2*lev*(abs(Im(pr))^2+(abs(Re(pr))+n)^2))); print();
      print(GREEN(`RAMANUJAN SERIES`)); print();
      if Im(z0)<>0 then
        print();
        print(GREEN(COMPLEX)); print();
      end if;
      vz0:=evalf(z0,40);
      if abs(vz0)>1 then
        print(R=infinity); print();
      else
        print(R=1/Pi); print();
      end if;
      print(GREEN(`HYPERGEOMETRIC FORM`));
  #
      print(); print(simplify(DELTA*RH)=1/Pi);
      print();
    end if;
  #
  else
    print(CONTA,opM);
    print(GREEN(`SPECIAL VALUE OF z`)=z0); print();
  end if;
  #
  CONTA:=CONTA+1; print();
  print(GREEN(`primitive multiplier`)=primim0,GREEN(`      primitive degree`)=abs(primim0)^(-2),
  GREEN(`     primitive tau`)=primitau);
  print(GREEN(`sequence of possible possible degrees for n=0,1,2,... corresponding to`)+delta=1);
  if Im(vz0)=0 then
    print(fullsimplify(lev*(Im(primitau)^2+(abs(Re(primitau))+n)^2)));
  end if;
  if Im(vz0)<>0 then
    print(Re(primitau)^2*n^2+Im(primitau)^2, n=0,1,2...);
  end if;
  print();
  print(GREEN(SECONDS = time() - o));
  print();
  print(RED(`************************************************************************************
  ****`));                                                                                   end:
  #
>
> # RAMANUJAN SERIES #
```

```
>
> RamaPi:=proc(lev,dd,degroot,polclass)
  #
  local o;
  global x,ro,fun,funsimp,Cz0,degz0,jjj,www,i,j,A,G,SOL,nn,uv0,AA,B,C,div,z0,vz0,J0,
  u0,v0,DISC,T,P0,P1,P2,rulepol,rulepoli,Lzvu0,M,L,LLzvu0,dT,dv,Ddv,ddv,atu0,Ddv0,dv0,ddv0,s,M0,D0,
  d0,
  aalpha0,bbeta0,nalpha0,nbeta0,cocalbe,primim0,primitau,cp0,bb0,tes,CONTA,opM,polcl;
  CONTA:=1; polcl:=polclass; print(GREEN(polclass));
  with(polytools);
  o := time();
  # Ddv0:=simplify(evala(subs({u=u0,v(u)=v0},simplify(Ddv,rulepol))),rulepol);
  if polcl=WEBER or polcl=RUSSELL then
  if lev<>1 and lev<>2 and lev<>3 and lev<>4 then
  print(RED(`level must be integer in [1,2,3,4]`));
  return();
  end if:
  #
  if lev <>2 and lev<>3 and lev<>4 and polclass=RUSSELL then
  print(GREEN(`For that level use WEBER (with capital letters)`)); return();
  end if;
  #
  if lev <>1 and lev<>2 and lev<>4 and polclass=WEBER then
  print(GREEN(`For that level use RUSSELL (with capital letters)`)); return();
  end if;
  if polcl = WEBER then webermodequ(lev, dd); end if:
  if polcl = RUSSELL then russellmodequ(lev,dd); end if:
  #                                 #
  T := u -> P(u, v(u)); dT:=diff(T(u),u);
  dv:=solve(dT,diff(v(u),u)); Ddv:=coeff(dT,diff(v(u),u));
  dv := simplify(factor(solve(dT,diff(v(u),u))));
  ddv := simplify(factor(subs(diff(v(u),u)=dv,diff(dv, u))));
  print();
  o:=time();
  #
  if polcl=WEBER then
     if lev=1 then ro:=1728: fun:=x->1728*x^12/(x^12-16)^3: end if:
     if lev=2 then ro:=256; fun:=x->-256*x^12/(x^12-64)^2; end if;
     if lev=4 then ro:=64: fun:=x->x^12/64: end if;
  end if;
  #
  if polcl=RUSSELL then
     if lev=2 then ro:=256: end if:
     if lev=3 then ro:=108; end if;
     if lev=4 then ro:=64: end if;
     fun:=x->4*x^h;
  end if;
  o:=time();
  #
  G := solve({P(u, v), fun(u)-fun(v)}, {u, v});
  nn := nops({G}); M:={}; L:={};
  for j from 1 to nn do
    Lzvu0:=simplify(evala(convert(G[j],radical)));
    LLzvu0:=subs({_Z=0,RootOf=0},Lzvu0);
    if LLzvu0<>Lzvu0 then
      Lzvu0:={v=0,u=0,z=0}
    else
    end if;
    v0:=op(2,Lzvu0[1]); u0:=op(2,Lzvu0[2]);
    z0:=simplify(evala(fun(u0))); vz0:=evalf(z0,100);
    atu0 := {u = u0, diff(v(u), u) = dv0, diff(v(u), u, u) = ddv0, v(u) = v0};
    P1 := evala(Minpoly(u0, x));
    P0 := evala(Minpoly(z0, x)); degz0:=degree(P0);
    P2 := evala(Minpoly(v0, x));
    rulepol := {subs(x = u, P1), subs(x = v(u), P2)};
    Ddv0:=simplify(evala(subs({u=u0,v(u)=v0},simplify(Ddv,rulepol))),rulepol);
    if (z=z0) in M or z0=0 or z0=1 then
    else
      if degz0=degroot then M:={z=z0} union M; L:= {{u=u0,v=v0,z=z0}} union L; end if;
    end if;
  end do;
  print(GREEN(`SPECIAL (OR SINGULAR) VALUES OF z`)); print(); opM:=nops(M);
  for j from 1 to nops(M) do print(z[0]=op(2,M[j])); end do;
  print(); print(GREEN(SECONDS = time() - o)); print();
  print(RED(________________________________________________________________));
  print(); print();
  else end if;
  for j from 1 to nops(L) do
     z0:=op(2,L[j][3]); u0:=op(2,L[j][1]); v0:=op(2,L[j][2]);
     atu0 := {u = u0, diff(v(u), u) = dv0, diff(v(u), u, u) = ddv0, v(u) = v0};
     P1 := evala(Minpoly(u0, x));
     P0:= evala(Minpoly(z0, x)); degz0:=degree(P0);
     P2 := evala(Minpoly(v0, x));
     rulepol := {subs(x = u, P1), subs(x = v(u), P2)};
     rulepoli := {subs(x = u, P1), subs(x = v, P2)};
     alias(po=pochhammer);
     s:=Pi/arcsin(sqrt(lev)/2):
     vz0:=evalf(z0,300);
     aalpha0:=evalf(solve(4*x*(1-x)=vz0,x)[1],250);
```

```
    bbeta0:=evalf(solve(4*x*(1-x)=vz0,x)[2],250);
    nalpha0:=evalf(hypergeom([1/s,1-1/s],[1],aalpha0),200);
    nbeta0:=evalf(hypergeom([1/s,1-1/s],[1],bbeta0),200);
    cocalbe:=round(evalf(abs(nalpha0/nbeta0)^(-2),180));
    cp0:=identify(evalf(simplify(nalpha0/nbeta0),180));
    primim0:=sqrt(Re(cp0)^2)+I*sqrt(Im(cp0)^2);
    M0:=primim0; D0:=abs(M0)^(-2); d0:=dd;
    primitau:=expand(simplify(I/abs(primim0^2)*primim0/sqrt(lev)));
    bb0:=evala(simplify(Im(primitau)*2*sqrt(1-z0)));
    mba(lev,dd,polclass);
  end do;
  end:
```

```
> aboutthis();
```

AUTOMATIC PROOFS OF RAMANUJAN-TYPE SERIES

(using modular Polynomials)

Jesús Guillera

University of Zaragoza

Written in 2023-24

REFERENCE: — The fastest series for 1/pi due to Ramanujan. Proofs by modular polynomials

(10)

```
> acknowledgements();
```


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ALIN BOSTAN — for his great idea of simplifying using an algebraic rule

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JORGE ZÚÑIGA — for checking this program with many examples and detecting an error that had ocurred very few times

(11)

```
> tryproc();
```

USE OF MAIN PROCEDURE

RamaPi(level=1,2,3,4, odd prime, degree roots=1,2,3,4, RUSSELL or WEBER)

RUSSELL for levels 2,3,4; WEBER for levels 1,2,4

(12)

```
> RamaPi(1,5,1,WEBER);
  RamaPi(1,7,1,WEBER);
  RamaPi(1,11,1,WEBER);
  RamaPi(1,17,1,WEBER);
  RamaPi(1,41,1,WEBER);
```

WEBER

LEVEL = 1,   DEGREE = 5

MODULAR EQUATION

$$\alpha(1-\alpha) = \frac{432\,u^{12}}{(u^{12}-16)^3}, \beta(1-\beta) = \frac{432\,v^{12}}{(v^{12}-16)^3}, \quad P(u,v)=0,$$

$$P(u,v) = u\,v\,(u^2 v^2 - 4)^2 - (u^3 + v^3)^2$$

SECONDS = .32e-1

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{27}{512}$$

$$z_0 = -\frac{1}{512}$$

$$z_0 = \frac{8}{1331}$$

SECONDS = 18.891

1, 3

SPECIAL VALUE of z = $\frac{8}{1331}$

multiplier = $\frac{2}{5} - \frac{I}{5}$, tau = $1 + 2\,I$, degree = 5

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$n^2 + 4$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6\,n)!\left(\frac{84\sqrt{33}\,n}{121} + \frac{20\sqrt{33}}{363}\right)}{(3\,n)!\,n!^3\,287496^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{20\sqrt{33}\ \mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{68}{63}\right], \left[\frac{5}{63}, 1, 1\right], \frac{8}{1331}\right)}{363} = \frac{1}{\pi}$$

primitive multiplier = $\frac{1}{2}$, primitive degree = 4, primitive tau = $2\,I$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$n^2 + 4$

SECONDS = .438

****

2, 3

SPECIAL VALUE of z = $-\frac{27}{512}$

multiplier = $\frac{\sqrt{11}}{10} - \frac{3\,I}{10}$, tau = $\frac{I\sqrt{11}}{2} + \frac{3}{2}$, degree = 5

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$n^2 + n + 3$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6\,n)!\left(\frac{77\sqrt{2}\,n}{32} + \frac{15\sqrt{2}}{64}\right)}{(3\,n)!\,n!^3\,(-32768)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{15\sqrt{2}\ \mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{169}{154}\right], \left[\frac{15}{154}, 1, 1\right], -\frac{27}{512}\right)}{64} = \frac{1}{\pi}$$

primitive multiplier = $\frac{\sqrt{11}}{6} + \frac{I}{6}$, primitive degree = 3, primitive tau = $-\frac{1}{2} + \frac{I\sqrt{11}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$n^2+n+3$$

SECONDS = 3.219

****

3, 3

SPECIAL VALUE of $z=-\frac{1}{512}$

multiplier $=\frac{\sqrt{19}}{10}+\frac{I}{10}$,   tau $=-\frac{1}{2}+\frac{I\sqrt{19}}{2}$,   degree $=5$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$n^2+n+5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(6\,n)!\left(\frac{57\sqrt{6}\,n}{32}+\frac{25\sqrt{6}}{192}\right)}{(3\,n)!\,n!^3\,(-884736)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{25\sqrt{6}\ \mathrm{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2},\frac{5}{6},\frac{367}{342}\right],\left[\frac{25}{342},1,1\right],-\frac{1}{512}\right)}{192}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{19}}{10}+\frac{I}{10}$,   primitive degree $=5$,   primitive tau $=-\frac{1}{2}+\frac{I\sqrt{19}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$n^2+n+5$$

SECONDS = 5.984

****

WEBER

LEVEL = 1,   DEGREE = 7

MODULAR EQUATION

$$\alpha\,(1-\alpha)=\frac{432\,u^{12}}{(u^{12}-16)^3},\ \beta\,(1-\beta)=\frac{432\,v^{12}}{(v^{12}-16)^3},\ \ P(u,v)=0,$$

$$P(u,v)=u\,v\,(u^3\,v^3+8)^2-(u^4+7\,u^2\,v^2+v^4)^2$$

SECONDS = .141

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0=-\frac{64}{125}$$

$$z_0=-\frac{9}{64000}$$

$$z_0=-\frac{1}{512}$$

$$z_0 = \frac{4}{125}$$

$$z_0 = \frac{64}{614125}$$

SECONDS = 21.391

1, 5

SPECIAL VALUE of z $= -\frac{64}{125}$

multiplier $= \frac{\sqrt{7}}{7}$, tau $= \frac{I}{2}\sqrt{7}$, degree $= 7$

$$\delta = \frac{1}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$4n^2 + 7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6n)!\left(\frac{21\sqrt{15}\,n}{25} + \frac{8\sqrt{15}}{75}\right)}{(3n)!\,n!^3\,(-3375)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{8\sqrt{15}\ \mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{71}{63}\right], \left[\frac{8}{63}, 1, 1\right], -\frac{64}{125}\right)}{75} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{7}}{4} + \frac{I}{4}$, primitive degree $= 2$, primitive tau $= \frac{I\sqrt{7}}{2} - \frac{1}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$n^2 + n + 2$$

SECONDS = .984

****

2, 5

SPECIAL VALUE of z $= \frac{64}{614125}$

multiplier $= \frac{\sqrt{7}}{7}$, tau $= I\sqrt{7}$, degree $= 7$

$$\delta = 1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$n^2 + 7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6n)!\left(\frac{2394\sqrt{255}\,n}{7225} + \frac{144\sqrt{255}}{7225}\right)}{(3n)!\,n!^3\,16581375^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{144\sqrt{255}\ \mathrm{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2},\frac{5}{6},\frac{141}{133}\right],\left[\frac{8}{133},1,1\right],\frac{64}{614125}\right)}{7225}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{7}}{7}$, primitive degree $=7$, primitive tau $=\mathrm{I}\sqrt{7}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta=1$

$$n^2+7$$

SECONDS = .296

****

primitive multiplier $=\frac{3\sqrt{3}}{14}+\frac{\mathrm{I}}{14}$, primitive degree $=7$, primitive tau $=-\frac{1}{2}+\frac{3\,\mathrm{I}\sqrt{3}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta=1$

$$n^2+n+7$$

SECONDS = 290.234

****

4, 5

SPECIAL VALUE of $z=\frac{4}{125}$

multiplier $=\frac{\sqrt{3}}{7}-\frac{2\,\mathrm{I}}{7}$, tau $=2+\mathrm{I}\sqrt{3}$, degree $=7$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$n^2+3$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(6\,n)!\left(\frac{22\sqrt{15}\,n}{25}+\frac{2\sqrt{15}}{25}\right)}{(3\,n)!\,n!^3\,54000^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{2\sqrt{15}\ \mathrm{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2},\frac{5}{6},\frac{12}{11}\right],\left[\frac{1}{11},1,1\right],\frac{4}{125}\right)}{25}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{3}}{3}$, primitive degree $=3$, primitive tau $=\mathrm{I}\sqrt{3}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta=1$

$$n^2+3$$

SECONDS = .500

****

5, 5

SPECIAL VALUE of $z=-\frac{1}{512}$

multiplier $=\frac{\sqrt{19}}{14}+\frac{3\,\mathrm{I}}{14}$, tau $=\frac{\mathrm{I}\sqrt{19}}{2}-\frac{3}{2}$, degree $=7$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$n^2+n+5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6\,n)!\left(\frac{57\sqrt{6}\,n}{32}+\frac{25\sqrt{6}}{192}\right)}{(3\,n)!\,n!^3\,(-884736)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{25\sqrt{6}\ \mathrm{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2},\frac{5}{6},\frac{367}{342}\right],\left[\frac{25}{342},1,1\right],-\frac{1}{512}\right)}{192}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{19}}{10}+\frac{I}{10}$, primitive degree $=5$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{19}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$n^2+n+5$$

SECONDS = 15.000

****

WEBER

LEVEL = 1, DEGREE = 11

MODULAR EQUATION

$$\alpha\,(1-\alpha)=\frac{432\,u^{12}}{(u^{12}-16)^3},\ \beta\,(1-\beta)=\frac{432\,v^{12}}{(v^{12}-16)^3},\quad P(u,v)=0,$$

$$P(u,v)=u\,v\,(u^5\,v^5-11\,u^4\,v^4+44\,u^3\,v^3-88\,u^2\,v^2+88\,u\,v-32)^2-(u^6+v^6)^2$$

SECONDS = .297

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0=-\frac{64}{125}$$

$$z_0=-\frac{27}{512}$$

$$z_0=-\frac{1}{512}$$

$$z_0=-\frac{1}{512000}$$

$$z_0=\frac{27}{125}$$

$$z_0=\frac{64}{614125}$$

SECONDS = 50.016

SPECIAL VALUE of $z = -\frac{64}{125}$

multiplier $= \frac{\sqrt{7}}{11} - \frac{2 I}{11}$, tau $= \frac{I\sqrt{7}}{2} + 1$, degree $= 11$

$\delta = \frac{1}{2}$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$4 n^2 + 7$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6n)!\left(\frac{21\sqrt{15}\,n}{25} + \frac{8\sqrt{15}}{75}\right)}{(3n)!\,n!^3\,(-3375)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{8\sqrt{15}\ \mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{71}{63}\right], \left[\frac{8}{63}, 1, 1\right], -\frac{64}{125}\right)}{75} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{7}}{4} + \frac{I}{4}$, primitive degree $= 2$, primitive tau $= \frac{I\sqrt{7}}{2} - \frac{1}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$n^2 + n + 2$

SECONDS = 2.766

****

2, 6

SPECIAL VALUE of $z = \frac{64}{614125}$

multiplier $= \frac{\sqrt{7}}{11} - \frac{2 I}{11}$, tau $= I\sqrt{7} + 2$, degree $= 11$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$n^2 + 7$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6n)!\left(\frac{2394\sqrt{255}\,n}{7225} + \frac{144\sqrt{255}}{7225}\right)}{(3n)!\,n!^3\,16581375^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{144\sqrt{255}\ \mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{141}{133}\right], \left[\frac{8}{133}, 1, 1\right], \frac{64}{614125}\right)}{7225} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{7}}{7}$, primitive degree $= 7$, primitive tau $= I\sqrt{7}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$n^2 + 7$

SECONDS = 1.515

****

3, 6

SPECIAL VALUE of $z = \frac{27}{125}$

multiplier $= \frac{3\,I}{11} + \frac{\sqrt{2}}{11}$,   tau $= -3 + I\sqrt{2}$,   degree $= 11$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$n^2 + 2$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6\,n)!\left(\frac{28\sqrt{5}\,n}{25} + \frac{3\sqrt{5}}{25}\right)}{(3\,n)!\,n!^3\,8000^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3\sqrt{5}\ \mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{31}{28}\right], \left[\frac{3}{28}, 1, 1\right], \frac{27}{125}\right)}{25} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{2}}{2}$,   primitive degree $= 2$,   primitive tau $= I\sqrt{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$n^2 + 2$$

SECONDS = 3.891

****

4, 6

SPECIAL VALUE of $z = -\frac{1}{512000}$

multiplier $= \frac{\sqrt{43}}{22} - \frac{I}{22}$,   tau $= \frac{I\sqrt{43}}{2} + \frac{1}{2}$,   degree $= 11$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$n^2 + n + 11$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6\,n)!\left(\frac{2709\sqrt{15}\,n}{1600} + \frac{263\sqrt{15}}{3200}\right)}{(3\,n)!\,n!^3\,(-884736000)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{263\sqrt{15}\ \mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{5681}{5418}\right], \left[\frac{263}{5418}, 1, 1\right], -\frac{1}{512000}\right)}{3200} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{43}}{22} + \frac{I}{22}$,   primitive degree $= 11$,   primitive tau $= -\frac{1}{2} + \frac{I\sqrt{43}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$n^2 + n + 11$$

SECONDS = 14.438

****

5, 6

SPECIAL VALUE of $z = -\frac{27}{512}$

multiplier $= \frac{\sqrt{11}}{11}$, tau $= \frac{I}{2}\sqrt{11}$, degree $= 11$

$\delta = \frac{1}{2}$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$4n^2 + 11$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6n)!\left(\frac{77\sqrt{2}\,n}{32} + \frac{15\sqrt{2}}{64}\right)}{(3n)!\,n!^3\,(-32768)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{15\sqrt{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{169}{154}\right], \left[\frac{15}{154}, 1, 1\right], -\frac{27}{512}\right)}{64} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{11}}{6} + \frac{I}{6}$, primitive degree $= 3$, primitive tau $= -\frac{1}{2} + \frac{I\sqrt{11}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$n^2 + n + 3$

SECONDS = 13.703

****

6, 6

SPECIAL VALUE of $z = -\frac{1}{512}$

multiplier $= \frac{\sqrt{19}}{22} - \frac{5\,I}{22}$, tau $= \frac{I\sqrt{19}}{2} + \frac{5}{2}$, degree $= 11$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$n^2 + n + 5$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6n)!\left(\frac{57\sqrt{6}\,n}{32} + \frac{25\sqrt{6}}{192}\right)}{(3n)!\,n!^3\,(-884736)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{25\sqrt{6}\,\mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{367}{342}\right], \left[\frac{25}{342}, 1, 1\right], -\frac{1}{512}\right)}{192} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{19}}{10} + \frac{I}{10}$, primitive degree $= 5$, primitive tau $= -\frac{1}{2} + \frac{I\sqrt{19}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$n^2 + n + 5$

SECONDS = 6.265

****

WEBER

LEVEL = 1, DEGREE = 17

MODULAR EQUATION

$$\alpha(1-\alpha) = \frac{432\,u^{12}}{(u^{12}-16)^3},\ \beta(1-\beta) = \frac{432\,v^{12}}{(v^{12}-16)^3},\quad P(u,v)=0,$$

$$P(u,v) = u\,v\,(u^8 v^8 - 34\,u^6 v^6 + 34\,u^7 v - 340\,u^4 v^4 + 34\,u\,v^7 - 544\,u^2 v^2 + 256)^2 - (17\,u^8 v^5 + 17\,u^5 v^8 + u^9 + 119\,u^6 v^3 + 119\,u^3 v^6 + v^9 + 272\,u^4 v + 272\,u\,v^4)^2$$

SECONDS = .578

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{1}{512}$$

$$z_0 = -\frac{1}{512000}$$

$$z_0 = -\frac{1}{85184000}$$

$$z_0 = \frac{8}{1331}$$

$$z_0 = \frac{27}{125}$$

SECONDS = 201.813

1, 5

SPECIAL VALUE of z $= \frac{27}{125}$

multiplier $= -\frac{3\,I}{17} + \frac{2\sqrt{2}}{17}$, tau $= \frac{3}{2} + I\sqrt{2}$, degree = 17

$$\delta = \frac{1}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\,\delta$

$$4\,n^2 + 4\,n + 9$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6\,n)!\left(\frac{28\sqrt{5}\,n}{25} + \frac{3\sqrt{5}}{25}\right)}{(3\,n)!\,n!^3\,8000^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3\sqrt{5}\ \mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{31}{28}\right], \left[\frac{3}{28}, 1, 1\right], \frac{27}{125}\right)}{25} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{2}}{2}$, primitive degree $= 2$, primitive tau $= I\sqrt{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$$n^2 + 2$$

SECONDS = 5.422

****

2, 5

SPECIAL VALUE of $z = \frac{8}{1331}$

multiplier $= \frac{4}{17} + \frac{I}{17}$, tau $= -\frac{1}{2} + 2I$, degree $= 17$

$$\delta = \frac{1}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$4n^2 + 4n + 17$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6n)!\left(\frac{84\sqrt{33}\,n}{121} + \frac{20\sqrt{33}}{363}\right)}{(3n)!\,n!^3\,287496^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{20\sqrt{33}\ \mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{68}{63}\right], \left[\frac{5}{63}, 1, 1\right], \frac{8}{1331}\right)}{363} = \frac{1}{\pi}$$

primitive multiplier $= \frac{1}{2}$, primitive degree $= 4$, primitive tau $= 2I$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$$n^2 + 4$$

SECONDS = 11.203

****

3, 5

SPECIAL VALUE of $z = -\frac{1}{85184000}$

multiplier $= \frac{\sqrt{67}}{34} - \frac{I}{34}$, tau $= \frac{I\sqrt{67}}{2} + \frac{1}{2}$, degree $= 17$

$$\delta = 1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$n^2 + n + 17$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6n)!\left(\frac{43617\sqrt{330}\,n}{96800} + \frac{10177\sqrt{330}}{580800}\right)}{(3n)!\,n!^3\,(-147197952000)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{10177\sqrt{330}\ \mathrm{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2},\frac{5}{6},\frac{271879}{261702}\right],\left[\frac{10177}{261702},1,1\right],-\frac{1}{85184000}\right)}{580800}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{67}}{34}+\frac{I}{34}$, primitive degree $=17$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{67}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta=1$

$$n^2+n+17$$

SECONDS = 6.562

****

4, 5

SPECIAL VALUE of z $=-\frac{1}{512000}$

multiplier $=\frac{\sqrt{43}}{34}-\frac{5\,I}{34}$, tau $=\frac{I\sqrt{43}}{2}+\frac{5}{2}$, degree $=17$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$n^2+n+11$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(6\,n)!\left(\frac{2709\sqrt{15}\,n}{1600}+\frac{263\sqrt{15}}{3200}\right)}{(3\,n)!\,n!^3\,(-884736000)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{263\sqrt{15}\ \mathrm{hypergeom}\left(\left[\frac{1}{6},\frac{1}{2},\frac{5}{6},\frac{5681}{5418}\right],\left[\frac{263}{5418},1,1\right],-\frac{1}{512000}\right)}{3200}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{43}}{22}+\frac{I}{22}$, primitive degree $=11$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{43}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta=1$

$$n^2+n+11$$

SECONDS = 3.609

****

5, 5

SPECIAL VALUE of z $=-\frac{1}{512}$

multiplier $=\frac{\sqrt{19}}{34}+\frac{7\,I}{34}$, tau $=\frac{I\sqrt{19}}{2}-\frac{7}{2}$, degree $=17$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$n^2+n+5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(6\,n)!\left(\frac{57\sqrt{6}\,n}{32}+\frac{25\sqrt{6}}{192}\right)}{(3\,n)!\,n!^3\,(-884736)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{25\sqrt{6}\ \mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{367}{342}\right], \left[\frac{25}{342}, 1, 1\right], -\frac{1}{512}\right)}{192} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{19}}{10} + \frac{I}{10}$, primitive degree $= 5$, primitive tau $= -\frac{1}{2} + \frac{I\sqrt{19}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$n^2 + n + 5$$

SECONDS = 14.906

****

WEBER

LEVEL = 1, DEGREE = 41

MODULAR EQUATION

$$\alpha(1-\alpha) = \frac{432\,u^{12}}{(u^{12}-16)^3},\ \beta(1-\beta) = \frac{432\,v^{12}}{(v^{12}-16)^3},\quad P(u,v)=0,$$

$$\begin{aligned}P(u,v) = u\,v\,\big(&u^{20} v^{20} - 574 u^{18} v^{18} + 4059 u^{19} v^{13} + 2050 u^{16} v^{16} + 4059 u^{13} v^{19} - 41 u^{20} v^{8} + 160310 u^{17} v^{11} + 462726 u^{14} v^{14} + 160310 u^{11} v^{17} \\ &- 41 u^{8} v^{20} + 155554 u^{18} v^{6} + 1753160 u^{15} v^{9} + 3571756 u^{12} v^{12} + 1753160 u^{9} v^{15} + 155554 u^{6} v^{18} - 574 u^{19} v - 701100 u^{16} v^{4} \\ &- 20156994 u^{13} v^{7} - 45567400 u^{10} v^{10} - 20156994 u^{7} v^{13} - 701100 u^{4} v^{16} - 574 u v^{19} + 2488864 u^{14} v^{2} + 28050560 u^{11} v^{5} \\ &+ 57148096 u^{8} v^{8} + 28050560 u^{5} v^{11} + 2488864 u^{2} v^{14} - 10496 u^{12} + 41039360 u^{9} v^{3} + 118457856 u^{6} v^{6} + 41039360 u^{3} v^{9} - 10496 v^{12} \\ &+ 16625664 u^{7} v + 8396800 u^{4} v^{4} + 16625664 u v^{7} - 37617664 u^{2} v^{2} + 1048576\big)^2 - \big(123 u^{20} v^{17} + 123 u^{17} v^{20} + 3772 u^{18} v^{15} \\ &+ 3772 u^{15} v^{18} + 40713 u^{19} v^{10} + 339111 u^{16} v^{13} + 339111 u^{13} v^{16} + 40713 u^{10} v^{19} + 943 u^{20} v^{5} + 733531 u^{17} v^{8} + 3112310 u^{14} v^{11} \\ &+ 3112310 u^{11} v^{14} + 733531 u^{8} v^{17} + 943 u^{5} v^{20} + u^{21} + 72939 u^{18} v^{3} - 6494359 u^{15} v^{6} - 36004437 u^{12} v^{9} - 36004437 u^{9} v^{12} \\ &- 6494359 u^{6} v^{15} + 72939 u^{3} v^{18} + v^{21} + 15088 u^{16} v + 11736496 u^{13} v^{4} + 49796960 u^{10} v^{7} + 49796960 u^{7} v^{10} + 11736496 u^{4} v^{13} \\ &+ 15088 u v^{16} + 10422528 u^{11} v^{2} + 86812416 u^{8} v^{5} + 86812416 u^{5} v^{8} + 10422528 u^{2} v^{11} + 15450112 u^{6} v^{3} + 15450112 u^{3} v^{6} \\ &+ 8060928 u^{4} v + 8060928 u v^{4}\big)^2\end{aligned}$$

SECONDS = 256.687

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{1}{512000}$$

$$z_0 = -\frac{1}{151931373056000}$$

$$z_0 = \frac{8}{1331}$$

$$z_0 = \frac{27}{125}$$

SECONDS = 3308.516

1, 4

SPECIAL VALUE of $z = \frac{27}{125}$

multiplier $= -\frac{3I}{41} + \frac{4\sqrt{2}}{41}$, tau $= \frac{3}{4} + I\sqrt{2}$, degree $= 41$

$$\delta = \frac{1}{4}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$16\,n^2 + 24\,n + 41$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6\,n)!\left(\frac{28\sqrt{5}\,n}{25} + \frac{3\sqrt{5}}{25}\right)}{(3\,n)!\,n!^3\,8000^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3\sqrt{5}\,\mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{31}{28}\right], \left[\frac{3}{28}, 1, 1\right], \frac{27}{125}\right)}{25} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{2}}{2}$, primitive degree $= 2$, primitive tau $= I\sqrt{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$n^2 + 2$$

SECONDS = 13.516

****

2, 4

SPECIAL VALUE of z $= \frac{8}{1331}$

multiplier $= \frac{4}{41} + \frac{5\,I}{41}$, tau $= -\frac{5}{2} + 2\,I$, degree $= 41$

$$\delta = \frac{1}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$4\,n^2 + 4\,n + 17$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6\,n)!\left(\frac{84\sqrt{33}\,n}{121} + \frac{20\sqrt{33}}{363}\right)}{(3\,n)!\,n!^3\,287496^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{20\sqrt{33}\,\mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{68}{63}\right], \left[\frac{5}{63}, 1, 1\right], \frac{8}{1331}\right)}{363} = \frac{1}{\pi}$$

primitive multiplier $= \frac{1}{2}$, primitive degree $= 4$, primitive tau $= 2\,I$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$n^2 + 4$$

SECONDS = 23.953

****

3, 4

SPECIAL VALUE of z $= -\frac{1}{151931373056000}$

multiplier $= \frac{\sqrt{163}}{82} - \frac{I}{82}$, tau $= \frac{I\sqrt{163}}{2} + \frac{1}{2}$, degree $= 41$

$$\delta = 1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$n^2 + n + 41$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6\,n)!\left(\frac{90856689\sqrt{10005}\,n}{711822400} + \frac{13591409\sqrt{10005}}{4270934400}\right)}{(3\,n)!\,n!^3\,(-262537412640768000)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{13591409\sqrt{10005}\ \mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{558731543}{545140134}\right], \left[\frac{13591409}{545140134}, 1, 1\right], -\frac{1}{151931373056000}\right)}{4270934400} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{163}}{82} + \frac{I}{82}$, primitive degree $= 41$, primitive tau $= -\frac{1}{2} + \frac{I\sqrt{163}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$n^2 + n + 41$$

SECONDS = 18.281

****

4, 4

SPECIAL VALUE of z $= -\frac{1}{512000}$

multiplier $= \frac{\sqrt{43}}{82} + \frac{11\,I}{82}$, tau $= \frac{I\sqrt{43}}{2} - \frac{11}{2}$, degree $= 41$

$$\delta = 1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$n^2 + n + 11$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(6\,n)!\left(\frac{2709\sqrt{15}\,n}{1600} + \frac{263\sqrt{15}}{3200}\right)}{(3\,n)!\,n!^3\,(-884736000)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{263\sqrt{15}\ \mathrm{hypergeom}\left(\left[\frac{1}{6}, \frac{1}{2}, \frac{5}{6}, \frac{5681}{5418}\right], \left[\frac{263}{5418}, 1, 1\right], -\frac{1}{512000}\right)}{3200} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{43}}{22} + \frac{I}{22}$, primitive degree $= 11$, primitive tau $= -\frac{1}{2} + \frac{I\sqrt{43}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$n^2 + n + 11$$

SECONDS = 28.438

*** (13)

```
> RamaPi(2,3,1,RUSSELL);
```

RUSSELL

LEVEL = 2,   DEGREE = 3

MODULAR EQUATION

$u = \alpha\beta, \quad v = (1-\alpha)(1-\beta), \quad P(u,v)=0,$

$P(u,v) = -u^4 + 388 u^3 v - 37638 u^2 v^2 + 388 u v^3 - v^4 + 4 u^3 + 2812 u^2 v + 2812 u v^2 + 4 v^3 - 6 u^2 + 892 u v - 6 v^2 + 4 u + 4 v - 1$

SECONDS = 2.079

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{1}{4}$$

$$z_0 = \frac{1}{9}$$

SECONDS = 0.

1, 2

SPECIAL VALUE of $z = -\frac{1}{4}$

multiplier $= \frac{\sqrt{10}}{6} - \frac{I\sqrt{2}}{6}$, tau $= \frac{I\sqrt{5}}{2} + \frac{1}{2}$, degree = 3

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$2 n^2 + 2 n + 3$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)! \left(\frac{5n}{2} + \frac{3}{8}\right)}{n!^4 (-1024)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3 \,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{23}{20}\right], \left[\frac{3}{20}, 1, 1\right], -\frac{1}{4}\right)}{8} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{10}}{6} + \frac{I\sqrt{2}}{6}$, primitive degree = 3, primitive tau $= -\frac{1}{2} + \frac{I\sqrt{5}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$2 n^2 + 2 n + 3$

SECONDS = .297

****

2, 2

SPECIAL VALUE of $z = \frac{1}{9}$

multiplier $= \frac{\sqrt{3}}{3}$, tau $= \frac{I}{2}\sqrt{6}$, degree = 3

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$2 n^2 + 3$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4 n)! \left(\frac{4\sqrt{3}\, n}{3} + \frac{\sqrt{3}}{6} \right)}{n!^4\, 2304^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{9}{8} \right], \left[\frac{1}{8}, 1, 1 \right], \frac{1}{9} \right)}{6} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{3}}{3}$, primitive degree $= 3$, primitive tau $= \frac{I}{2}\sqrt{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2 n^2 + 3$$

SECONDS = .203

** **(14)**

```
> RamaPi(2,3,1,RUSSELL);
```

RUSSELL

LEVEL = 2, DEGREE = 3

MODULAR EQUATION

$$u = \alpha \beta, \quad v = (1-\alpha)(1-\beta), \quad P(u,v)=0,$$

$$P(u,v) = -u^4 + 388 u^3 v - 37638 u^2 v^2 + 388 u v^3 - v^4 + 4 u^3 + 2812 u^2 v + 2812 u v^2 + 4 v^3 - 6 u^2 + 892 u v - 6 v^2 + 4 u + 4 v - 1$$

SECONDS = 7.453

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{1}{4}$$

$$z_0 = \frac{1}{9}$$

SECONDS = .31e-1

1, 2

SPECIAL VALUE of z $= -\frac{1}{4}$

multiplier $= \frac{\sqrt{10}}{6} - \frac{I\sqrt{2}}{6}$, tau $= \frac{I\sqrt{5}}{2} + \frac{1}{2}$, degree $= 3$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$2 n^2 + 2 n + 3$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{5\,n}{2}+\frac{3}{8}\right)}{n!^4\,(-1024)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{23}{20}\right],\left[\frac{3}{20},1,1\right],-\frac{1}{4}\right)}{8}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{10}}{6}+\frac{I\sqrt{2}}{6}$, primitive degree $=3$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{5}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$2\,n^2+2\,n+3$

SECONDS = .234

****

2, 2

SPECIAL VALUE of z $=\frac{1}{9}$

multiplier $=\frac{\sqrt{3}}{3}$, tau $=\frac{I}{2}\sqrt{6}$, degree $=3$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$2\,n^2+3$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{4\sqrt{3}\,n}{3}+\frac{\sqrt{3}}{6}\right)}{n!^4\,2304^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{9}{8}\right],\left[\frac{1}{8},1,1\right],\frac{1}{9}\right)}{6}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{3}}{3}$, primitive degree $=3$, primitive tau $=\frac{I}{2}\sqrt{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$2\,n^2+3$

SECONDS = .219

*** **(15)**

```
> RamaPi(2,5,1,WEBER);
  RamaPi(2,7,1,WEBER);
  RamaPi(2,11,1,WEBER);
  RamaPi(2,13,1,WEBER);
  RamaPi(2,19,1,WEBER);
  RamaPi(2,29,1,WEBER);
  RamaPi(2,5,1,RUSSELL);
  RamaPi(2,7,1,RUSSELL);
  RamaPi(2,11,1,RUSSELL);
  RamaPi(2,29,1,RUSSELL);
```

WEBER

LEVEL = 2, DEGREE = 5

MODULAR EQUATION

$$\alpha(1-\alpha) = -\frac{256\,u^{12}}{(-u^{12}+64)^2},\ \beta(1-\beta) = -\frac{256\,v^{12}}{(-v^{12}+64)^2},\quad P(u,v)=0,$$

$$P(u,v) = u\,v\,(u^2 v^2 - 4)^2 - (u^3 + v^3)^2$$

SECONDS = .62e-1

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{1}{4}$$

$$z_0 = -\frac{1}{48}$$

$$z_0 = \frac{1}{9}$$

$$z_0 = \frac{1}{81}$$

$$z_0 = \frac{32}{81}$$

SECONDS = 4.672

1, 5

SPECIAL VALUE of $z = -\frac{1}{4}$

multiplier $= \frac{\sqrt{5}}{5}$, tau $= \frac{I}{2}\sqrt{5}$, degree $= 5$

$$\delta = \frac{\sqrt{2}}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$4\,n^2 + 5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{5\,n}{2}+\frac{3}{8}\right)}{n!^4\,(-1024)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{23}{20}\right],\left[\frac{3}{20},1,1\right],-\frac{1}{4}\right)}{8} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{10}}{6} + \frac{I\sqrt{2}}{6}$, primitive degree $= 3$, primitive tau $= \frac{I\sqrt{5}}{2} - \frac{1}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2\,n^2 + 2\,n + 3$$

SECONDS = .406

****

2, 5

SPECIAL VALUE of $z = \frac{32}{81}$

multiplier $= \frac{2}{5} - \frac{I}{5}$, tau $= \frac{1}{2} + I$, degree $= 5$

$\delta = \frac{\sqrt{2}}{2}$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$4 n^2 + 4 n + 5$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4 n)! \left(\frac{14 n}{9} + \frac{2}{9}\right)}{n!^4\, 648^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{2 \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{8}{7}\right], \left[\frac{1}{7}, 1, 1\right], \frac{32}{81}\right)}{9} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{2}}{2}$, primitive degree $= 2$, primitive tau $= I$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$2 n^2 + 2$

SECONDS = .328

****

3, 5

SPECIAL VALUE of $z = \frac{1}{81}$

multiplier $= \frac{\sqrt{5}}{5}$, tau $= \frac{I}{2}\sqrt{10}$, degree $= 5$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$2 n^2 + 5$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4 n)! \left(\frac{20 \sqrt{2}\, n}{9} + \frac{2 \sqrt{2}}{9}\right)}{n!^4\, 20736^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{2 \sqrt{2} \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{11}{10}\right], \left[\frac{1}{10}, 1, 1\right], \frac{1}{81}\right)}{9} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{5}}{5}$, primitive degree $= 5$, primitive tau $= \frac{I}{2}\sqrt{10}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$2\,n^2+5$$

SECONDS = .375

****

4, 5

SPECIAL VALUE of $z=\frac{1}{9}$

multiplier $=\frac{\sqrt{3}}{5}-\frac{I\sqrt{2}}{5}$, tau $=\frac{I\sqrt{6}}{2}+1$, degree $=5$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$2\,n^2+3$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(4\,n)!\left(\frac{4\sqrt{3}\,n}{3}+\frac{\sqrt{3}}{6}\right)}{n!^4\,2304^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{9}{8}\right],\left[\frac{1}{8},1,1\right],\frac{1}{9}\right)}{6}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{3}}{3}$, primitive degree $=3$, primitive tau $=\frac{I}{2}\sqrt{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$2\,n^2+3$$

SECONDS = .969

****

5, 5

SPECIAL VALUE of $z=-\frac{1}{48}$

multiplier $=\frac{3\sqrt{2}}{10}+\frac{I\sqrt{2}}{10}$, tau $=-\frac{1}{2}+\frac{3\,I}{2}$, degree $=5$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$2\,n^2+2\,n+5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(4\,n)!\left(\frac{7\sqrt{3}\,n}{4}+\frac{3\sqrt{3}}{16}\right)}{n!^4\,(-12288)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{31}{28}\right],\left[\frac{3}{28},1,1\right],-\frac{1}{48}\right)}{16}=\frac{1}{\pi}$$

primitive multiplier $= \frac{3\sqrt{2}}{10} + \frac{I\sqrt{2}}{10}$, primitive degree $=5$, primitive tau $= -\frac{1}{2} + \frac{3\,I}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$$2\,n^2 + 2\,n + 5$$

SECONDS = .844

****

WEBER

LEVEL = 2, DEGREE = 7

MODULAR EQUATION

$$\alpha\,(1-\alpha) = -\frac{256\,u^{12}}{(-u^{12}+64)^2},\ \beta\,(1-\beta) = -\frac{256\,v^{12}}{(-v^{12}+64)^2},\ P(u,v)=0,$$

$$P(u,v) = u\,v\,(u^3\,v^3+8)^2 - (u^4+7\,u^2\,v^2+v^4)^2$$

SECONDS = .109

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{256}{3969}$$

$$z_0 = -\frac{16}{9}$$

$$z_0 = -\frac{1}{4}$$

$$z_0 = -\frac{1}{324}$$

$$z_0 = \frac{1}{9}$$

$$z_0 = \frac{1}{81}$$

$$z_0 = \frac{256}{81}$$

SECONDS = 7.484

1, 7

SPECIAL VALUE of z $= -\frac{256}{3969}$

multiplier $= \frac{\sqrt{7}}{7}$, tau $= \frac{I}{2}\sqrt{7}$, degree $=7$

$$\delta = \frac{\sqrt{2}}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$4\,n^2 + 7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{65\sqrt{7}\,n}{63}+\frac{8\sqrt{7}}{63}\right)}{n!^4\,(-3969)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{8\sqrt{7}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{73}{65}\right],\left[\frac{8}{65},1,1\right],-\frac{256}{3969}\right)}{63}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{14}}{8}+\frac{I\sqrt{2}}{8}$, primitive degree $=4$, primitive tau $=\frac{I\sqrt{7}}{2}-\frac{1}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta=1$

$$2\,n^2+2\,n+4$$

SECONDS = .328

****

2, 7

SPECIAL VALUE of z $=-\frac{1}{324}$

multiplier $=\frac{\sqrt{26}}{14}+\frac{I\sqrt{2}}{14}$, tau $=-\frac{1}{2}+\frac{I\sqrt{13}}{2}$, degree $=7$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$2\,n^2+2\,n+7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{65\,n}{18}+\frac{23}{72}\right)}{n!^4\,(-82944)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{23\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{283}{260}\right],\left[\frac{23}{260},1,1\right],-\frac{1}{324}\right)}{72}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{26}}{14}+\frac{I\sqrt{2}}{14}$, primitive degree $=7$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{13}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta=1$

$$2\,n^2+2\,n+7$$

SECONDS = .734

****

3, 7

SPECIAL VALUE of z $=-\frac{1}{4}$

multiplier $=\frac{\sqrt{10}}{14}+\frac{3\,I\sqrt{2}}{14}$, tau $=\frac{I\sqrt{5}}{2}-\frac{3}{2}$, degree $=7$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$2\,n^2+2\,n+3$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{5n}{2}+\frac{3}{8}\right)}{n!^4\,(-1024)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3\,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{23}{20}\right], \left[\frac{3}{20}, 1, 1\right], -\frac{1}{4}\right)}{8} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{10}}{6} + \frac{I\sqrt{2}}{6}$, primitive degree $= 3$, primitive tau $= \frac{I\sqrt{5}}{2} - \frac{1}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$$2n^2 + 2n + 3$$

SECONDS = .641

****

4, 7

SPECIAL VALUE of z $= \frac{1}{81}$

multiplier $= \frac{\sqrt{5}}{7} - \frac{I\sqrt{2}}{7}$, tau $= \frac{I\sqrt{10}}{2} + 1$, degree $= 7$

$$\delta = 1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$2n^2 + 5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{20\sqrt{2}\,n}{9}+\frac{2\sqrt{2}}{9}\right)}{n!^4\,20736^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{2\sqrt{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{11}{10}\right], \left[\frac{1}{10}, 1, 1\right], \frac{1}{81}\right)}{9} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{5}}{5}$, primitive degree $= 5$, primitive tau $= \frac{I}{2}\sqrt{10}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$$2n^2 + 5$$

SECONDS = 3.110

****

5, 7

SPECIAL VALUE of z $= \frac{1}{9}$

multiplier $= \frac{\sqrt{6}}{7} - \frac{I}{7}$, tau $= \frac{I\sqrt{6}}{2} + \frac{1}{2}$, degree $= 7$

$$\delta = \frac{\sqrt{2}}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$4n^2 + 4n + 7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{4\sqrt{3}\,n}{3}+\frac{\sqrt{3}}{6}\right)}{n!^4\,2304^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{9}{8}\right],\left[\frac{1}{8},1,1\right],\frac{1}{9}\right)}{6}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{3}}{3}$, primitive degree $=3$, primitive tau $=\frac{I}{2}\sqrt{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\,\delta=1$

$$2\,n^2+3$$

SECONDS = .907

****

6, 7

SPECIAL VALUE of $z=\frac{256}{81}$

multiplier $=\frac{\sqrt{7}}{7}$, tau $=\frac{I}{4}\sqrt{7}$, degree $=7$

$$\delta=\frac{\sqrt{2}}{4}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\,\delta$

$$16\,n^2+7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{35\,I\,n}{18}+\frac{4\,I}{9}\right)}{n!^4\,81^n}=\infty$$

HYPERGEOMETRIC FORM

$$\frac{4\,I}{9}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{43}{35}\right],\left[\frac{8}{35},1,1\right],\frac{256}{81}\right)=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{14}}{4}+\frac{I\sqrt{2}}{4}$, primitive degree $=1$, primitive tau $=-\frac{1}{4}+\frac{I\sqrt{7}}{4}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\,\delta=1$

$$2\,n^2+n+1$$

SECONDS = .547

****

7, 7

SPECIAL VALUE of $z=-\frac{16}{9}$

multiplier $=\frac{\sqrt{3}}{7}-\frac{2\,I}{7}$, tau $=\frac{I\sqrt{3}}{2}+1$, degree $=7$

$$\delta=\frac{\sqrt{2}}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$4n^2+3$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{5\sqrt{3}\,n}{3}+\frac{\sqrt{3}}{3}\right)}{n!^4\,(-144)^n} = \infty$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{6}{5}\right],\left[\frac{1}{5},1,1\right],-\frac{16}{9}\right)}{3} = \frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{6}}{4}+\frac{I\sqrt{2}}{4}$, primitive degree $=2$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{3}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$2n^2+2n+2$$

SECONDS = .484

****

WEBER

LEVEL = 2, DEGREE = 11

MODULAR EQUATION

$$\alpha(1-\alpha) = -\frac{256u^{12}}{(-u^{12}+64)^2},\ \beta(1-\beta) = -\frac{256v^{12}}{(-v^{12}+64)^2},\ P(u,v)=0,$$

$$P(u,v) = u\,v\,(u^5v^5-11u^4v^4+44u^3v^3-88u^2v^2+88uv-32)^2-(u^6+v^6)^2$$

SECONDS = .187

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{256}{3969}$$

$$z_0 = -\frac{1}{324}$$

$$z_0 = \frac{1}{9}$$

$$z_0 = \frac{1}{81}$$

$$z_0 = \frac{1}{2401}$$

$$z_0 = \frac{1}{9801}$$

$$z_0 = \frac{256}{81}$$

SECONDS = 15.390

1, 7

SPECIAL VALUE of $z=-\frac{256}{3969}$

multiplier $=\frac{\sqrt{7}}{11}-\frac{2\,I}{11}$, tau $=\frac{I\sqrt{7}}{2}+1$, degree $=11$

$$\delta=\frac{\sqrt{2}}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\,\delta$

$$4\,n^2+7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(4\,n)!\left(\frac{65\sqrt{7}\,n}{63}+\frac{8\sqrt{7}}{63}\right)}{n!^4\,(-3969)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{8\sqrt{7}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{73}{65}\right],\left[\frac{8}{65},1,1\right],-\frac{256}{3969}\right)}{63}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{14}}{8}+\frac{I\sqrt{2}}{8}$, primitive degree $=4$, primitive tau $=\frac{I\sqrt{7}}{2}-\frac{1}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\,\delta=1$

$$2\,n^2+2\,n+4$$

SECONDS = .485

****

2, 7

SPECIAL VALUE of $z=-\frac{1}{324}$

multiplier $=\frac{\sqrt{26}}{22}-\frac{3\,I\sqrt{2}}{22}$, tau $=\frac{I\sqrt{13}}{2}+\frac{3}{2}$, degree $=11$

$$\delta=1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\,\delta$

$$2\,n^2+2\,n+7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(4\,n)!\left(\frac{65\,n}{18}+\frac{23}{72}\right)}{n!^4\,(-82944)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{23\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{283}{260}\right],\left[\frac{23}{260},1,1\right],-\frac{1}{324}\right)}{72}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{26}}{14}+\frac{I\sqrt{2}}{14}$, primitive degree $=7$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{13}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\,\delta=1$

$$2\,n^2+2\,n+7$$

SECONDS = .594

****

3, 7

SPECIAL VALUE of z $= \frac{1}{9801}$

multiplier $= \frac{\sqrt{11}}{11}$, tau $= \frac{I}{2}\sqrt{22}$, degree = 11

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$2\,n^2 + 11$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{140\sqrt{11}\,n}{99} + \frac{19\sqrt{11}}{198}\right)}{n!^4\ 2509056^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{19\sqrt{11}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{299}{280}\right], \left[\frac{19}{280}, 1, 1\right], \frac{1}{9801}\right)}{198} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{11}}{11}$, primitive degree = 11, primitive tau $= \frac{I}{2}\sqrt{22}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$2\,n^2 + 11$

SECONDS = .453

****

4, 7

SPECIAL VALUE of z $= \frac{1}{81}$

multiplier $= \frac{\sqrt{10}}{11} - \frac{I}{11}$, tau $= \frac{I\sqrt{10}}{2} + \frac{1}{2}$, degree = 11

$\delta = \frac{\sqrt{2}}{2}$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$4\,n^2 + 4\,n + 11$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{20\sqrt{2}\,n}{9} + \frac{2\sqrt{2}}{9}\right)}{n!^4\ 20736^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{2\sqrt{2}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{11}{10}\right], \left[\frac{1}{10}, 1, 1\right], \frac{1}{81}\right)}{9} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{5}}{5}$, primitive degree = 5, primitive tau $= \frac{I}{2}\sqrt{10}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$$2\,n^2+5$$

SECONDS = 6.969

****

5, 7

SPECIAL VALUE of $z=\frac{1}{9}$

$$\text{multiplier}=\frac{\sqrt{3}}{11}-\frac{2\,I\sqrt{2}}{11},\quad \text{tau}=\frac{I\sqrt{6}}{2}+2,\quad \text{degree}=11$$

$$\delta=1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\,\delta$

$$2\,n^2+3$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(4\,n)!\left(\frac{4\sqrt{3}\,n}{3}+\frac{\sqrt{3}}{6}\right)}{n!^4\,2304^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{9}{8}\right],\left[\frac{1}{8},1,1\right],\frac{1}{9}\right)}{6}=\frac{1}{\pi}$$

$$\text{primitive multiplier}=\frac{\sqrt{3}}{3},\quad \text{primitive degree}=3,\quad \text{primitive tau}=\frac{I}{2}\sqrt{6}$$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\,\delta=1$

$$2\,n^2+3$$

SECONDS = .781

****

6, 7

SPECIAL VALUE of $z=\frac{256}{81}$

$$\text{multiplier}=\frac{\sqrt{7}}{11}+\frac{2\,I}{11},\quad \text{tau}=\frac{I\sqrt{7}}{4}-\frac{1}{2},\quad \text{degree}=11$$

$$\delta=\frac{\sqrt{2}}{4}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\,\delta$

$$16\,n^2+16\,n+11$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(4\,n)!\left(\frac{35\,I\,n}{18}+\frac{4\,I}{9}\right)}{n!^4\,81^n}=\infty$$

HYPERGEOMETRIC FORM

$$\frac{4\,I}{9}\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{43}{35}\right],\left[\frac{8}{35},1,1\right],\frac{256}{81}\right)=\frac{1}{\pi}$$

$$\text{primitive multiplier}=\frac{\sqrt{14}}{4}+\frac{I\sqrt{2}}{4},\quad \text{primitive degree}=1,\quad \text{primitive tau}=-\frac{1}{4}+\frac{I\sqrt{7}}{4}$$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$2n^2+n+1$$

SECONDS = 93.093

****

$$7, 7$$

SPECIAL VALUE of z $=\frac{1}{2401}$

$$\text{multiplier}=-\frac{I\sqrt{2}}{11}+\frac{3}{11}, \quad \text{tau}=1+\frac{3I\sqrt{2}}{2}, \quad \text{degree}=11$$

$$\delta=1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$2n^2+9$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(4n)!\left(\frac{120\sqrt{3}\,n}{49}+\frac{9\sqrt{3}}{49}\right)}{n!^4\,614656^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{9\sqrt{3}\,\text{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{43}{40}\right],\left[\frac{3}{40},1,1\right],\frac{1}{2401}\right)}{49}=\frac{1}{\pi}$$

$$\text{primitive multiplier}=\frac{1}{3}, \quad \text{primitive degree}=9, \quad \text{primitive tau}=\frac{3I}{2}\sqrt{2}$$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$2n^2+9$$

SECONDS = 11.734

****

WEBER

LEVEL $=2$, DEGREE $=13$

MODULAR EQUATION

$$\alpha(1-\alpha)=-\frac{256u^{12}}{(-u^{12}+64)^2}, \beta(1-\beta)=-\frac{256v^{12}}{(-v^{12}+64)^2}, \quad P(u,v)=0,$$

$$P(u,v)=uv(u^6v^6-64)^2-(u^7+13u^6v+52u^5v^2+78u^4v^3+78u^3v^4+52u^2v^5+13uv^6+v^7)^2$$

SECONDS = .328

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0=-\frac{16}{9}$$

$$z_0=-\frac{1}{48}$$

$$z_0=-\frac{1}{324}$$

$$z_0 = -\frac{1}{25920}$$

$$z_0 = \frac{1}{81}$$

$$z_0 = \frac{1}{9801}$$

$$z_0 = \frac{32}{81}$$

SECONDS = 19.359

1, 7

SPECIAL VALUE of $z = -\frac{16}{9}$

multiplier $= \frac{I}{13} + \frac{2\sqrt{3}}{13}$, tau $= -\frac{1}{4} + \frac{I\sqrt{3}}{2}$, degree $= 13$

$$\delta = \frac{\sqrt{2}}{4}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$16 n^2 + 8 n + 13$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{5\sqrt{3}\,n}{3} + \frac{\sqrt{3}}{3}\right)}{n!^4\,(-144)^n} = \infty$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{6}{5}\right], \left[\frac{1}{5}, 1, 1\right], -\frac{16}{9}\right)}{3} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{6}}{4} + \frac{I\sqrt{2}}{4}$, primitive degree $= 2$, primitive tau $= -\frac{1}{2} + \frac{I\sqrt{3}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2 n^2 + 2 n + 2$$

SECONDS = .812

****

2, 7

SPECIAL VALUE of $z = -\frac{1}{324}$

multiplier $= \frac{\sqrt{13}}{13}$, tau $= \frac{I}{2}\sqrt{13}$, degree $= 13$

$$\delta = \frac{\sqrt{2}}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$4 n^2 + 13$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{65\,n}{18}+\frac{23}{72}\right)}{n!^4\,(-82944)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{23\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{283}{260}\right],\left[\frac{23}{260},1,1\right],-\frac{1}{324}\right)}{72}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{26}}{14}+\frac{I\sqrt{2}}{14}$, primitive degree $=7$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{13}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta=1$

$$2\,n^2+2\,n+7$$

SECONDS = .813

****

3, 7

SPECIAL VALUE of z $=\frac{1}{9801}$

multiplier $=\frac{\sqrt{11}}{13}-\frac{I\sqrt{2}}{13}$, tau $=\frac{I\sqrt{22}}{2}+1$, degree $=13$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$2\,n^2+11$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{140\sqrt{11}\,n}{99}+\frac{19\sqrt{11}}{198}\right)}{n!^4\,2509056^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{19\sqrt{11}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{299}{280}\right],\left[\frac{19}{280},1,1\right],\frac{1}{9801}\right)}{198}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{11}}{11}$, primitive degree $=11$, primitive tau $=\frac{I}{2}\sqrt{22}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta=1$

$$2\,n^2+11$$

SECONDS = 9.843

****

4, 7

SPECIAL VALUE of z $=\frac{32}{81}$

multiplier $=\frac{2}{13}-\frac{3\,I}{13}$, tau $=\frac{3}{2}+I$, degree $=13$

$\delta=\frac{\sqrt{2}}{2}$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$4\,n^2+4\,n+5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{14\,n}{9}+\frac{2}{9}\right)}{n!^4\,648^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{2\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{8}{7}\right],\left[\frac{1}{7},1,1\right],\frac{32}{81}\right)}{9}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{2}}{2}$, primitive degree $=2$, primitive tau $=\mathrm{I}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta=1$

$$2\,n^2+2$$

SECONDS = .282

****

5, 7

SPECIAL VALUE of z $=\frac{1}{81}$

multiplier $=\frac{\sqrt{5}}{13}-\frac{2\,\mathrm{I}\sqrt{2}}{13}$, tau $=\frac{\mathrm{I}\sqrt{10}}{2}+2$, degree $=13$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$2\,n^2+5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{20\sqrt{2}\,n}{9}+\frac{2\sqrt{2}}{9}\right)}{n!^4\,20736^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{2\sqrt{2}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{11}{10}\right],\left[\frac{1}{10},1,1\right],\frac{1}{81}\right)}{9}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{5}}{5}$, primitive degree $=5$, primitive tau $=\frac{\mathrm{I}}{2}\sqrt{10}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta=1$

$$2\,n^2+5$$

SECONDS = 11.719

****

6, 7

SPECIAL VALUE of z $=-\frac{1}{25920}$

multiplier $=\frac{5\sqrt{2}}{26}+\frac{\mathrm{I}\sqrt{2}}{26}$, tau $=-\frac{1}{2}+\frac{5\,\mathrm{I}}{2}$, degree $=13$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$2\,n^2+2\,n+13$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)! \left(\frac{161\sqrt{5}\,n}{72} + \frac{41\sqrt{5}}{288} \right)}{n!^4 \, (-6635520)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{41\sqrt{5}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{685}{644} \right], \left[\frac{41}{644}, 1, 1 \right], -\frac{1}{25920} \right)}{288} = \frac{1}{\pi}$$

primitive multiplier $= \frac{5\sqrt{2}}{26} + \frac{I\sqrt{2}}{26}$, primitive degree $= 13$, primitive tau $= -\frac{1}{2} + \frac{5\,I}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2n^2 + 2n + 13$$

SECONDS = 9.875

****

7, 7

SPECIAL VALUE of $z = -\frac{1}{48}$

multiplier $= \frac{3}{13} - \frac{2\,I}{13}$, tau $= 1 + \frac{3\,I}{2}$, degree $= 13$

$$\delta = \frac{\sqrt{2}}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$4n^2 + 9$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)! \left(\frac{7\sqrt{3}\,n}{4} + \frac{3\sqrt{3}}{16} \right)}{n!^4 \, (-12288)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{31}{28} \right], \left[\frac{3}{28}, 1, 1 \right], -\frac{1}{48} \right)}{16} = \frac{1}{\pi}$$

primitive multiplier $= \frac{3\sqrt{2}}{10} + \frac{I\sqrt{2}}{10}$, primitive degree $= 5$, primitive tau $= -\frac{1}{2} + \frac{3\,I}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2n^2 + 2n + 5$$

SECONDS = 1.219

****

WEBER

LEVEL $= 2$, DEGREE $= 19$

MODULAR EQUATION

$$\alpha(1-\alpha) = -\frac{256\,u^{12}}{(-u^{12}+64)^2},\ \beta(1-\beta) = -\frac{256\,v^{12}}{(-v^{12}+64)^2},\quad P(u,v)=0,$$

$$P(u,v) = u\,v\,(u^9 v^9 + 95 v^4 u^8 + 38 u^6 v^6 + 95 u^4 v^8 - 760 u^5 v - 304 u^3 v^3 - 760 u v^5 - 512)^2 - (19 u^9 v^7 + 19 u^7 v^9 + u^{10} + 114 u^8 v^2 - 95 u^6 v^4 - 95 u^4 v^6 + 114 u^2 v^8 + v^{10} + 1216 u^3 v + 1216 u v^3)^2$$

SECONDS = .968

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{16}{9}$$

$$z_0 = -\frac{1}{324}$$

$$z_0 = -\frac{1}{777924}$$

$$z_0 = \frac{1}{81}$$

$$z_0 = \frac{1}{2401}$$

$$z_0 = \frac{1}{9801}$$

SECONDS = 29.562

1, 6

SPECIAL VALUE of $z = \frac{1}{2401}$

multiplier $= -\frac{I}{19} + \frac{3\sqrt{2}}{19}$, tau $= \frac{1}{2} + \frac{3\,I\sqrt{2}}{2}$, degree $= 19$

$$\delta = \frac{\sqrt{2}}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\,\delta$

$$4\,n^2 + 4\,n + 19$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{120\sqrt{3}\,n}{49} + \frac{9\sqrt{3}}{49}\right)}{n!^4\,614656^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{9\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{43}{40}\right], \left[\frac{3}{40}, 1, 1\right], \frac{1}{2401}\right)}{49} = \frac{1}{\pi}$$

primitive multiplier $= \frac{1}{3}$, primitive degree $= 9$, primitive tau $= \frac{3\,I}{2}\sqrt{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\,\delta = 1$

$$2\,n^2 + 9$$

SECONDS = 1.735

****

2, 6

SPECIAL VALUE of $z = -\frac{1}{777924}$

multiplier $= \frac{\sqrt{74}}{38} - \frac{I\sqrt{2}}{38}$, tau $= \frac{I\sqrt{37}}{2} + \frac{1}{2}$, degree $= 19$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$2n^2 + 2n + 19$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{5365n}{882} + \frac{1123}{3528}\right)}{n!^4(-199148544)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{1123\,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{22583}{21460}\right], \left[\frac{1123}{21460}, 1, 1\right], -\frac{1}{777924}\right)}{3528} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{74}}{38} + \frac{I\sqrt{2}}{38}$, primitive degree $= 19$, primitive tau $= -\frac{1}{2} + \frac{I\sqrt{37}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2n^2 + 2n + 19$$

SECONDS = 1.141

****

3, 6

SPECIAL VALUE of $z = -\frac{1}{324}$

multiplier $= \frac{\sqrt{26}}{38} + \frac{5I\sqrt{2}}{38}$, tau $= \frac{I\sqrt{13}}{2} - \frac{5}{2}$, degree $= 19$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$2n^2 + 2n + 7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{65n}{18} + \frac{23}{72}\right)}{n!^4(-82944)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{23\,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{283}{260}\right], \left[\frac{23}{260}, 1, 1\right], -\frac{1}{324}\right)}{72} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{26}}{14} + \frac{I\sqrt{2}}{14}$, primitive degree $= 7$, primitive tau $= -\frac{1}{2} + \frac{I\sqrt{13}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2n^2 + 2n + 7$$

SECONDS = .735

****

4, 6

SPECIAL VALUE of z $= \frac{1}{9801}$

multiplier $= \frac{\sqrt{11}}{19} - \frac{2\,I\sqrt{2}}{19}$, tau $= \frac{I\sqrt{22}}{2} + 2$, degree = 19

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$2n^2 + 11$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{140\sqrt{11}\,n}{99} + \frac{19\sqrt{11}}{198}\right)}{n!^4\, 2509056^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{19\sqrt{11}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{299}{280}\right], \left[\frac{19}{280}, 1, 1\right], \frac{1}{9801}\right)}{198} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{11}}{11}$, primitive degree = 11, primitive tau $= \frac{I}{2}\sqrt{22}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$2n^2 + 11$

SECONDS = 13.328

****

5, 6

SPECIAL VALUE of z $= \frac{1}{81}$

multiplier $= \frac{\sqrt{10}}{19} - \frac{3\,I}{19}$, tau $= \frac{I\sqrt{10}}{2} + \frac{3}{2}$, degree = 19

$\delta = \frac{\sqrt{2}}{2}$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$4n^2 + 4n + 11$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{20\sqrt{2}\,n}{9} + \frac{2\sqrt{2}}{9}\right)}{n!^4\, 20736^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{2\sqrt{2}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{11}{10}\right], \left[\frac{1}{10}, 1, 1\right], \frac{1}{81}\right)}{9} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{5}}{5}$, primitive degree = 5, primitive tau $= \frac{I}{2}\sqrt{10}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$2n^2+5$

SECONDS = 13.000

****

6, 6

SPECIAL VALUE of $z=-\frac{16}{9}$

multiplier $=\frac{\sqrt{3}}{19}-\frac{4\,I}{19}$, tau $=\frac{I\sqrt{3}}{2}+2$, degree $=19$

$\delta=\frac{\sqrt{2}}{2}$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\,\delta$

$4n^2+3$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(4n)!\left(\frac{5\sqrt{3}\,n}{3}+\frac{\sqrt{3}}{3}\right)}{n!^4\,(-144)^n}=\infty$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{6}{5}\right],\left[\frac{1}{5},1,1\right],-\frac{16}{9}\right)}{3}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{6}}{4}+\frac{I\sqrt{2}}{4}$, primitive degree $=2$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{3}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\,\delta=1$

$2n^2+2n+2$

SECONDS = .625

****

WEBER

LEVEL = 2, DEGREE = 29

MODULAR EQUATION

$$\alpha(1-\alpha)=-\frac{256\,u^{12}}{(-u^{12}+64)^2},\ \beta(1-\beta)=-\frac{256\,v^{12}}{(-v^{12}+64)^2},\quad P(u,v)=0,$$

$$P(u,v)=u\,v\,(u^{14}v^{14}-203\,u^{12}v^{12}-783\,u^{13}v^{7}+1334\,u^{10}v^{10}-783\,u^{7}v^{13}-29\,u^{14}v^{2}-12470\,u^{8}v^{8}-29\,u^{2}v^{14}+116\,u^{12}+49880\,u^{6}v^{6}+116\,v^{12}+50112\,u^{7}v-85376\,u^{4}v^{4}+50112\,u\,v^{7}+207872\,u^{2}v^{2}-16384)^2-(667\,u^{13}v^{10}+667\,u^{10}v^{13}+261\,u^{14}v^{5}-5713\,u^{11}v^{8}-5713\,u^{8}v^{11}+261\,u^{5}v^{14}+u^{15}+5365\,u^{12}v^{3}+37642\,u^{9}v^{6}+37642\,u^{6}v^{9}+5365\,u^{3}v^{12}+v^{15}+4176\,u^{10}v-91408\,u^{7}v^{4}-91408\,u^{4}v^{7}+4176\,u\,v^{10}+170752\,u^{5}v^{2}+170752\,u^{2}v^{5})^2$$

SECONDS = 18.515

SPECIAL (OR SINGULAR) VALUES OF z

$z_0=-\frac{256}{3969}$

$$z_0 = -\frac{1}{4}$$

$$z_0 = -\frac{1}{48}$$

$$z_0 = -\frac{1}{324}$$

$$z_0 = -\frac{1}{25920}$$

$$z_0 = \frac{1}{9}$$

$$z_0 = \frac{1}{9801}$$

$$z_0 = \frac{1}{96059601}$$

$$z_0 = \frac{32}{81}$$

$$z_0 = \frac{256}{81}$$

SECONDS = 33.328

1, 10

SPECIAL VALUE of $z = -\frac{256}{3969}$

multiplier $= -\frac{I}{29} + \frac{2\sqrt{7}}{29}$, tau $= \frac{1}{4} + \frac{I\sqrt{7}}{2}$, degree $= 29$

$$\delta = \frac{\sqrt{2}}{4}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$16 n^2 + 8 n + 29$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{65\sqrt{7}\,n}{63} + \frac{8\sqrt{7}}{63}\right)}{n!^4\,(-3969)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{8\sqrt{7}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{73}{65}\right], \left[\frac{8}{65}, 1, 1\right], -\frac{256}{3969}\right)}{63} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{14}}{8} + \frac{I\sqrt{2}}{8}$, primitive degree $= 4$, primitive tau $= \frac{I\sqrt{7}}{2} - \frac{1}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2 n^2 + 2 n + 4$$

SECONDS = 5.516

****

2, 10

SPECIAL VALUE of $z = -\frac{1}{324}$

multiplier $= \frac{\sqrt{13}}{29} - \frac{4\,I}{29}$, tau $= \frac{I\sqrt{13}}{2} + 2$, degree $= 29$

$$\delta = \frac{\sqrt{2}}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$4 n^2 + 13$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4 n)! \left(\frac{65 n}{18} + \frac{23}{72}\right)}{n!^4 (-82944)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{23 \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{283}{260}\right], \left[\frac{23}{260}, 1, 1\right], -\frac{1}{324}\right)}{72} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{26}}{14} + \frac{I\sqrt{2}}{14}$, primitive degree $=7$, primitive tau $= -\frac{1}{2} + \frac{I\sqrt{13}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2 n^2 + 2 n + 7$$

SECONDS = 21.203

****

3, 10

SPECIAL VALUE of $z = -\frac{1}{4}$

multiplier $= \frac{3 I}{29} + \frac{2\sqrt{5}}{29}$, tau $= -\frac{3}{4} + \frac{I\sqrt{5}}{2}$, degree $=29$

$$\delta = \frac{\sqrt{2}}{4}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$16 n^2 + 24 n + 29$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4 n)! \left(\frac{5 n}{2} + \frac{3}{8}\right)}{n!^4 (-1024)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3 \operatorname{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{23}{20}\right], \left[\frac{3}{20}, 1, 1\right], -\frac{1}{4}\right)}{8} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{10}}{6} + \frac{I\sqrt{2}}{6}$, primitive degree $=3$, primitive tau $= \frac{I\sqrt{5}}{2} - \frac{1}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2 n^2 + 2 n + 3$$

SECONDS = 13.610

****

4, 10

SPECIAL VALUE of $z = \frac{32}{81}$

multiplier $= \frac{2}{29} - \frac{5\,I}{29}$, tau $= \frac{5}{2} + I$, degree $= 29$

$\delta = \frac{\sqrt{2}}{2}$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$4\,n^2 + 4\,n + 5$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{14\,n}{9} + \frac{2}{9}\right)}{n!^4\,648^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{2\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{8}{7}\right], \left[\frac{1}{7}, 1, 1\right], \frac{32}{81}\right)}{9} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{2}}{2}$, primitive degree $= 2$, primitive tau $= I$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$2\,n^2 + 2$

SECONDS = 1.547

****

5, 10

SPECIAL VALUE of $z = \frac{1}{9801}$

multiplier $= \frac{\sqrt{11}}{29} - \frac{3\,I\sqrt{2}}{29}$, tau $= \frac{I\sqrt{22}}{2} + 3$, degree $= 29$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$2\,n^2 + 11$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{140\sqrt{11}\,n}{99} + \frac{19\sqrt{11}}{198}\right)}{n!^4\,2509056^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{19\sqrt{11}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{299}{280}\right], \left[\frac{19}{280}, 1, 1\right], \frac{1}{9801}\right)}{198} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{11}}{11}$, primitive degree $= 11$, primitive tau $= \frac{I}{2}\sqrt{22}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$2\,n^2 + 11$

SECONDS = 18.328

****

6, 10

SPECIAL VALUE of $z = \frac{1}{96059601}$

multiplier $= \frac{\sqrt{29}}{29}$, tau $= \frac{I}{2}\sqrt{58}$, degree $= 29$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$2n^2 + 29$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{52780\sqrt{2}\,n}{9801} + \frac{2206\sqrt{2}}{9801}\right)}{n!^4\, 24591257856^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{2206\sqrt{2}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{27493}{26390}\right], \left[\frac{1103}{26390}, 1, 1\right], \frac{1}{96059601}\right)}{9801} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{29}}{29}$, primitive degree $= 29$, primitive tau $= \frac{I}{2}\sqrt{58}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$2n^2 + 29$

SECONDS = .953

****

7, 10

SPECIAL VALUE of $z = \frac{1}{9}$

multiplier $= \frac{3\sqrt{3}}{29} - \frac{I\sqrt{2}}{29}$, tau $= \frac{I\sqrt{6}}{2} + \frac{1}{3}$, degree $= 29$

$\delta = \frac{1}{3}$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$18n^2 + 12n + 29$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{4\sqrt{3}\,n}{3} + \frac{\sqrt{3}}{6}\right)}{n!^4\, 2304^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{9}{8}\right], \left[\frac{1}{8}, 1, 1\right], \frac{1}{9}\right)}{6} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{3}}{3}$, primitive degree $= 3$, primitive tau $= \frac{I}{2}\sqrt{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$2n^2 + 3$

SECONDS = 228.812

****

8, 10

SPECIAL VALUE of $z = -\frac{1}{48}$

multiplier $= \frac{3\sqrt{2}}{58} + \frac{7\,I\sqrt{2}}{58}$, tau $= -\frac{7}{2} + \frac{3\,I}{2}$, degree $= 29$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$2\,n^2 + 2\,n + 5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{7\sqrt{3}\,n}{4} + \frac{3\sqrt{3}}{16}\right)}{n!^4\,(-12288)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{31}{28}\right], \left[\frac{3}{28}, 1, 1\right], -\frac{1}{48}\right)}{16} = \frac{1}{\pi}$$

primitive multiplier $= \frac{3\sqrt{2}}{10} + \frac{I\sqrt{2}}{10}$, primitive degree $= 5$, primitive tau $= -\frac{1}{2} + \frac{3\,I}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2\,n^2 + 2\,n + 5$$

SECONDS = 13.391

****

Error, (in property/ProbablyNonZero) cannot determine if this expression is true or false: ln(.1e-10*abs(6999711.3*D(v)(-1/2*2^(1/2)*(5^(1/2)-3))+.36757e6))/ln(10) < -6

RUSSELL

LEVEL $= 2$, DEGREE $= 5$

MODULAR EQUATION

$$u^2 = \alpha\,\beta,\quad v^2 = (1-\alpha)\,(1-\beta),\quad P(u,v)=0,$$
$$P(u,v) = u^3 + 323\,u^2\,v + 323\,u\,v^2 + v^3 - 3\,u^2 + 186\,u\,v - 3\,v^2 + 3\,u + 3\,v - 1$$

SECONDS = 1.953

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{1}{48}$$

$$z_0 = \frac{1}{9}$$

$$z_0 = \frac{1}{81}$$

SECONDS = .63e-1

1, 3

SPECIAL VALUE of $z = \frac{1}{9}$

multiplier $= \frac{\sqrt{3}}{5} - \frac{I\sqrt{2}}{5}$, tau $= \frac{I\sqrt{6}}{2} + 1$, degree $= 5$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$2n^2 + 3$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{4\sqrt{3}\,n}{3} + \frac{\sqrt{3}}{6}\right)}{n!^4\, 2304^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{9}{8}\right], \left[\frac{1}{8}, 1, 1\right], \frac{1}{9}\right)}{6} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{3}}{3}$, primitive degree $= 3$, primitive tau $= \frac{I}{2}\sqrt{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$2n^2 + 3$

SECONDS = .687

****

2, 3

SPECIAL VALUE of $z = \frac{1}{81}$

multiplier $= \frac{\sqrt{5}}{5}$, tau $= \frac{I}{2}\sqrt{10}$, degree $= 5$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$2n^2 + 5$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{20\sqrt{2}\,n}{9} + \frac{2\sqrt{2}}{9}\right)}{n!^4\, 20736^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{2\sqrt{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{11}{10}\right], \left[\frac{1}{10}, 1, 1\right], \frac{1}{81}\right)}{9} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{5}}{5}$, primitive degree $= 5$, primitive tau $= \frac{I}{2}\sqrt{10}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$2n^2 + 5$

SECONDS = .172

****

3, 3

SPECIAL VALUE of z $= -\frac{1}{48}$

multiplier $= \frac{3\sqrt{2}}{10} - \frac{I\sqrt{2}}{10}$, tau $= \frac{1}{2} + \frac{3\,I}{2}$, degree $= 5$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$2\,n^2 + 2\,n + 5$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{7\sqrt{3}\,n}{4} + \frac{3\sqrt{3}}{16}\right)}{n!^4\,(-12288)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{31}{28}\right], \left[\frac{3}{28}, 1, 1\right], -\frac{1}{48}\right)}{16} = \frac{1}{\pi}$$

primitive multiplier $= \frac{3\sqrt{2}}{10} + \frac{I\sqrt{2}}{10}$, primitive degree $= 5$, primitive tau $= -\frac{1}{2} + \frac{3\,I}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$2\,n^2 + 2\,n + 5$

SECONDS = .328

****

RUSSELL

LEVEL $= 2$, DEGREE $= 7$

MODULAR EQUATION

$u^4 = \alpha\,\beta$, $v^4 = (1-\alpha)\,(1-\beta)$, $P(u,v)=0$,

$P(u,v) = -u^4 + 88\,u^3\,v - 146\,u^2\,v^2 + 88\,u\,v^3 - v^4 + 2\,u^2 + 40\,u\,v + 2\,v^2 - 1$

SECONDS = 3.094

SPECIAL (OR SINGULAR) VALUES OF z

$z_0 = -\frac{1}{4}$

$z_0 = -\frac{1}{324}$

$z_0 = \frac{1}{81}$

SECONDS = .235

1, 3

SPECIAL VALUE of $z = \frac{1}{81}$

multiplier $= \frac{\sqrt{5}}{7} + \frac{I\sqrt{2}}{7}$, tau $= \frac{I\sqrt{10}}{2} - 1$, degree $= 7$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$2\,n^2 + 5$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{20\sqrt{2}\,n}{9} + \frac{2\sqrt{2}}{9}\right)}{n!^4\,20736^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{2\sqrt{2}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{11}{10}\right], \left[\frac{1}{10}, 1, 1\right], \frac{1}{81}\right)}{9} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{5}}{5}$, primitive degree $= 5$, primitive tau $= \frac{I}{2}\sqrt{10}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$2\,n^2 + 5$

SECONDS = 1.266

****

2, 3

SPECIAL VALUE of $z = -\frac{1}{4}$

multiplier $= \frac{\sqrt{10}}{14} + \frac{3\,I\sqrt{2}}{14}$, tau $= \frac{I\sqrt{5}}{2} - \frac{3}{2}$, degree $= 7$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$2\,n^2 + 2\,n + 3$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{5\,n}{2} + \frac{3}{8}\right)}{n!^4\,(-1024)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{23}{20}\right], \left[\frac{3}{20}, 1, 1\right], -\frac{1}{4}\right)}{8} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{10}}{6} + \frac{I\sqrt{2}}{6}$, primitive degree $= 3$, primitive tau $= \frac{I\sqrt{5}}{2} - \frac{1}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$2\,n^2 + 2\,n + 3$

SECONDS = .313

****

3, 3

SPECIAL VALUE of z $= -\frac{1}{324}$

multiplier $= \frac{\sqrt{26}}{14} - \frac{I\sqrt{2}}{14}$, tau $= \frac{I\sqrt{13}}{2} + \frac{1}{2}$, degree $= 7$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$2n^2 + 2n + 7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)!\left(\frac{65n}{18} + \frac{23}{72}\right)}{n!^4 (-82944)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{23\,\mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{283}{260}\right], \left[\frac{23}{260}, 1, 1\right], -\frac{1}{324}\right)}{72} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{26}}{14} + \frac{I\sqrt{2}}{14}$, primitive degree $= 7$, primitive tau $= -\frac{1}{2} + \frac{I\sqrt{13}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2n^2 + 2n + 7$$

SECONDS = .391

****

RUSSELL

LEVEL $= 2$, DEGREE $= 11$

MODULAR EQUATION

$u^4 = \alpha\beta$, $v^4 = (1-\alpha)(1-\beta)$, *P(u,v)=0,*

$$P(u,v) = u^6 + 1996u^5v - 3021u^4v^2 + 9176u^3v^3 - 3021u^2v^4 + 1996uv^5 + v^6 - 3u^4 + 1896u^3v - 6198u^2v^2 + 1896uv^3 - 3v^4 + 3u^2 + 204uv + 3v^2 - 1$$

SECONDS = 6.172

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{1}{324}$$

$$z_0 = \frac{1}{9}$$

$$z_0 = \frac{1}{2401}$$

$$z_0 = \frac{1}{9801}$$

SECONDS = .484

$$1, 4$$

SPECIAL VALUE of $z = \frac{1}{2401}$

multiplier $= \frac{3}{11} + \frac{I\sqrt{2}}{11}$, tau $= \frac{3 I\sqrt{2}}{2} - 1$, degree $= 11$

$$\delta = 1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$2 n^2 + 9$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)! \left(\frac{120\sqrt{3}\, n}{49} + \frac{9\sqrt{3}}{49}\right)}{n!^4\, 614656^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{9\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{43}{40}\right], \left[\frac{3}{40}, 1, 1\right], \frac{1}{2401}\right)}{49} = \frac{1}{\pi}$$

primitive multiplier $= \frac{1}{3}$, primitive degree $= 9$, primitive tau $= \frac{3 I}{2}\sqrt{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2 n^2 + 9$$

SECONDS = 1.766

****

$$2, 4$$

SPECIAL VALUE of $z = -\frac{1}{324}$

multiplier $= \frac{\sqrt{26}}{22} + \frac{3 I\sqrt{2}}{22}$, tau $= \frac{I\sqrt{13}}{2} - \frac{3}{2}$, degree $= 11$

$$\delta = 1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$2 n^2 + 2 n + 7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4n)! \left(\frac{65 n}{18} + \frac{23}{72}\right)}{n!^4 (-82944)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{23\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{283}{260}\right], \left[\frac{23}{260}, 1, 1\right], -\frac{1}{324}\right)}{72} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{26}}{14} + \frac{I\sqrt{2}}{14}$, primitive degree $= 7$, primitive tau $= -\frac{1}{2} + \frac{I\sqrt{13}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2 n^2 + 2 n + 7$$

SECONDS = .422

****

3, 4

SPECIAL VALUE of z = $\frac{1}{9}$

multiplier = $\frac{\sqrt{3}}{11} - \frac{2\,\mathrm{I}\sqrt{2}}{11}$, tau = $\frac{\mathrm{I}\sqrt{6}}{2} + 2$, degree = 11

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\,\delta$

$2\,n^2 + 3$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{4\sqrt{3}\,n}{3} + \frac{\sqrt{3}}{6}\right)}{n!^4\,2304^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{9}{8}\right], \left[\frac{1}{8}, 1, 1\right], \frac{1}{9}\right)}{6} = \frac{1}{\pi}$$

primitive multiplier = $\frac{\sqrt{3}}{3}$, primitive degree = 3, primitive tau = $\frac{\mathrm{I}}{2}\sqrt{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\,\delta = 1$

$2\,n^2 + 3$

SECONDS = 1.187

****

4, 4

SPECIAL VALUE of z = $\frac{1}{9801}$

multiplier = $\frac{\sqrt{11}}{11}$, tau = $\frac{\mathrm{I}}{2}\sqrt{22}$, degree = 11

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\,\delta$

$2\,n^2 + 11$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{140\sqrt{11}\,n}{99} + \frac{19\sqrt{11}}{198}\right)}{n!^4\,2509056^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{19\sqrt{11}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{299}{280}\right], \left[\frac{19}{280}, 1, 1\right], \frac{1}{9801}\right)}{198} = \frac{1}{\pi}$$

primitive multiplier = $\frac{\sqrt{11}}{11}$, primitive degree = 11, primitive tau = $\frac{\mathrm{I}}{2}\sqrt{22}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\,\delta = 1$

$2\,n^2 + 11$

SECONDS = .187

****

RUSSELL

LEVEL = 2, DEGREE = 29

MODULAR EQUATION

$$u^2 = \alpha\,\beta, \quad v^2 = (1-\alpha)\,(1-\beta), \quad P(u,v)=0,$$

$$\begin{aligned}
P(u,v) = {} & u^{15} + 7592191322114338383\,u^{14}\,v - 1396662256859735369480 7\,u^{13}\,v^2 + 984343976483719041652973 5\,u^{12}\,v^3 \\
& - 8805085619521769611971357 9\,u^{11}\,v^4 + 264881102454684464005109883\,u^{10}\,v^5 - 339110438936155583303166131\,u^9\,v^6 \\
& + 327822432596177883480907299\,u^8\,v^7 + 327822432596177883480907299\,u^7\,v^8 - 339110438936155583303166131\,u^6\,v^9 \\
& + 264881102454684464005109883\,u^5\,v^{10} - 88050856195217696119713579\,u^4\,v^{11} + 9843439764837190416529735\,u^3\,v^{12} \\
& - 139666225685973536948 07\,u^2\,v^{13} + 7592191322114338383\,u\,v^{14} + v^{15} - 15\,u^{14} + 44151100393296850926\,u^{13}\,v \\
& - 8011594661339095201288 5\,u^{12}\,v^2 + 8651254491779851942902828\,u^{11}\,v^3 - 4968768590703227018344592 7\,u^{10}\,v^4 \\
& + 73254121967733863952911 22\,u^9\,v^5 + 9127344839368205789360542 83\,u^8\,v^6 - 7497393276499302989094144 24\,u^7\,v^7 \\
& + 912734483936820578936054283\,u^6\,v^8 + 732541219677338639529112 2\,u^5\,v^9 - 49687685907032270183445927\,u^4\,v^{10} \\
& + 8651254491779851942902828\,u^3\,v^{11} - 80115946613390952012885\,u^2\,v^{12} + 44151100393296850926\,u\,v^{13} - 15\,v^{14} + 105\,u^{13} \\
& + 111337818333508248789\,u^{12}\,v - 746747209361822980929 30\,u^{11}\,v^2 - 13575976708893635268568626\,u^{10}\,v^3 \\
& + 175519360241020438387389123\,u^9\,v^4 - 225782350367252329130120 01\,u^8\,v^5 + 685632655975243084716955860\,u^7\,v^6 \\
& + 685632655975243084716955860\,u^6\,v^7 - 22578235036725232913012001\,u^5\,v^8 + 175519360241020438387389123\,u^4\,v^9 \\
& - 13575976708893635268568626\,u^3\,v^{10} - 74674720936182298092930\,u^2\,v^{11} + 111337818333508248789\,u\,v^{12} + 105\,v^{13} - 455\,u^{12} \\
& + 1595614871401157312 44\,u^{11}\,v + 1746957801043645207346 42\,u^{10}\,v^2 - 111018137411170324997838 12\,u^9\,v^3 \\
& + 201075954132759831412608055\,u^8\,v^4 + 349154723555304342205053272\,u^7\,v^5 + 120037255935309848803783612\,u^6\,v^6 \\
& + 349154723555304342205053272\,u^5\,v^7 + 201075954132759831412608055\,u^4\,v^8 - 11101813741117032499783812\,u^3\,v^9 \\
& + 174695780104364520734642\,u^2\,v^{10} + 159561487140115731244\,u\,v^{11} - 455\,v^{12} + 1365\,u^{11} + 143135164756761661287\,u^{10}\,v \\
& + 28382473555896967601357 1\,u^9\,v^2 + 2841283696852964184955593\,u^8\,v^3 - 56670680773339657795710 54\,u^7\,v^4 \\
& + 191014637062060944039908838\,u^6\,v^5 + 191014637062060944039908838\,u^5\,v^6 - 5667068077333965779571054\,u^4\,v^7 \\
& + 2841283696852964184955593\,u^3\,v^8 + 283824735558969676013571\,u^2\,v^9 + 143135164756761661287\,u\,v^{10} + 1365\,v^{11} - 3003\,u^{10} \\
& + 834850853140014440 82\,u^9\,v - 14149343647392627703263\,u^8\,v^2 + 19590560700558736521951 60\,u^7\,v^3 \\
& - 975516250436904707339636 22\,u^6\,v^4 - 465313755154527491729363 40\,u^5\,v^5 - 97551625043690470733963622\,u^4\,v^6 \\
& + 1959056070055873652195160\,u^3\,v^7 - 14149343647392627703263\,u^2\,v^8 + 83485085314001444082\,u\,v^9 - 3003\,v^{10} + 5005\,u^9 \\
& + 31881828039654884597\,u^8\,v - 1674907742798058055877 56\,u^7\,v^2 + 281991280871312184134125 2\,u^6\,v^3 \\
& - 2616731238291742368065167 4\,u^5\,v^4 - 26167312382917423680651674\,u^4\,v^5 + 2819912808713121841341252\,u^3\,v^6 \\
& - 16749077427980580558775 6\,u^2\,v^7 + 31881828039654884597\,u\,v^8 + 5005\,v^9 - 6435\,u^8 + 7844529197113035240\,u^7\,v \\
& - 3098895240787375296046 8\,u^6\,v^2 + 3116654341158724299627864\,u^5\,v^3 + 192469949160224866468567 8\,u^4\,v^4 \\
& + 3116654341158724299627864\,u^3\,v^5 - 30988952407873752960468\,u^2\,v^6 + 7844529197113035240\,u\,v^7 - 6435\,v^8 + 6435\,u^7 \\
& + 1196071300121674677\,u^6\,v + 20439274318981208511135\,u^5\,v^2 + 712650893196916561319241\,u^4\,v^3 \\
& + 712650893196916561319241\,u^3\,v^4 + 20439274318981208511135\,u^2\,v^5 + 1196071300121674677\,u\,v^6 + 6435\,v^7 - 5005\,u^6 \\
& + 105608490494850034\,u^5\,v - 1295519266509406265411\,u^4\,v^2 + 16061879206164618424444\,u^3\,v^3 - 1295519266509406265411\,u^2\,v^4 \\
& + 105608490494850034\,u\,v^5 - 5005\,v^6 + 3003\,u^5 + 4830887024610855\,u^4\,v + 141113497039864831 50\,u^3\,v^2 \\
& + 14111349703986483150\,u^2\,v^3 + 4830887024610855\,u\,v^4 + 3003\,v^5 - 1365\,u^4 + 94634825344812\,u^3\,v - 17548052640024318\,u^2\,v^2 \\
& + 94634825344812\,u\,v^3 - 1365\,v^4 + 455\,u^3 + 549304034965\,u^2\,v + 549304034965\,u\,v^2 + 455\,v^3 - 105\,u^2 + 368941806\,u\,v - 105\,v^2 \\
& + 15\,u + 15\,v - 1
\end{aligned}$$

SECONDS = 8473.250

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{1}{48}$$

$$z_0 = \frac{1}{9}$$

$$z_0 = \frac{1}{9801}$$

$$z_0 = \frac{1}{96059601}$$

SECONDS = .829

1, 4

SPECIAL VALUE of z $= \frac{1}{9}$

multiplier $= \frac{3\sqrt{3}}{29} - \frac{I\sqrt{2}}{29}$, tau $= \frac{I\sqrt{6}}{2} + \frac{1}{3}$, degree $= 29$

$$\delta = \frac{1}{3}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$18\,n^2 + 12\,n + 29$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{4\sqrt{3}\,n}{3} + \frac{\sqrt{3}}{6}\right)}{n!^4\,2304^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{9}{8}\right], \left[\frac{1}{8}, 1, 1\right], \frac{1}{9}\right)}{6} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{3}}{3}$, primitive degree $= 3$, primitive tau $= \frac{I}{2}\sqrt{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$2\,n^2 + 3$$

SECONDS = 3.266

****

2, 4

SPECIAL VALUE of z $= \frac{1}{9801}$

multiplier $= \frac{\sqrt{11}}{29} - \frac{3\,I\sqrt{2}}{29}$, tau $= \frac{I\sqrt{22}}{2} + 3$, degree $= 29$

$$\delta = 1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$2\,n^2 + 11$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{140\sqrt{11}\,n}{99} + \frac{19\sqrt{11}}{198}\right)}{n!^4\,2509056^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{19\sqrt{11}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{299}{280}\right],\left[\frac{19}{280},1,1\right],\frac{1}{9801}\right)}{198}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{11}}{11}$, primitive degree $=11$, primitive tau $=\frac{I}{2}\sqrt{22}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\,\delta=1$

$$2n^2+11$$

SECONDS = 4.203

****

3, 4

SPECIAL VALUE of z $=\frac{1}{96059601}$

multiplier $=\frac{\sqrt{29}}{29}$, tau $=\frac{I}{2}\sqrt{58}$, degree $=29$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\,\delta$

$$2n^2+29$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(4n)!\left(\frac{52780\sqrt{2}\,n}{9801}+\frac{2206\sqrt{2}}{9801}\right)}{n!^4\,24591257856^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{2206\sqrt{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{27493}{26390}\right],\left[\frac{1103}{26390},1,1\right],\frac{1}{96059601}\right)}{9801}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{29}}{29}$, primitive degree $=29$, primitive tau $=\frac{I}{2}\sqrt{58}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\,\delta=1$

$$2n^2+29$$

SECONDS = .266

****

4, 4

SPECIAL VALUE of z $=-\frac{1}{48}$

multiplier $=\frac{3\sqrt{2}}{58}+\frac{7I\sqrt{2}}{58}$, tau $=-\frac{7}{2}+\frac{3I}{2}$, degree $=29$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\,\delta$

$$2n^2+2n+5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(4n)!\left(\frac{7\sqrt{3}\,n}{4}+\frac{3\sqrt{3}}{16}\right)}{n!^4\,(-12288)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{31}{28}\right], \left[\frac{3}{28}, 1, 1\right], -\frac{1}{48}\right)}{16} = \frac{1}{\pi}$$

primitive multiplier $= \frac{3\sqrt{2}}{10} + \frac{I\sqrt{2}}{10}$, primitive degree $= 5$, primitive tau $= -\frac{1}{2} + \frac{3\,I}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$$2\,n^2 + 2\,n + 5$$

SECONDS = .406

*** **(16)**

```
> RamaPi(2, 3, 1, RUSSELL);
```

RUSSELL

LEVEL = 2, DEGREE = 3

MODULAR EQUATION

$$u = \alpha\,\beta,\quad v = (1-\alpha)\,(1-\beta),\quad P(u,v)=0,$$

$$P(u,v) = -u^4 + 388\,u^3\,v - 37638\,u^2\,v^2 + 388\,u\,v^3 - v^4 + 4\,u^3 + 2812\,u^2\,v + 2812\,u\,v^2 + 4\,v^3 - 6\,u^2 + 892\,u\,v - 6\,v^2 + 4\,u + 4\,v - 1$$

SECONDS = 2.140

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{1}{4}$$

$$z_0 = \frac{1}{9}$$

SECONDS = .16e-1

1, 2

SPECIAL VALUE of z $= -\frac{1}{4}$

multiplier $= \frac{\sqrt{10}}{6} - \frac{I\sqrt{2}}{6}$, tau $= \frac{I\sqrt{5}}{2} + \frac{1}{2}$, degree $= 3$

$$\delta = 1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$2\,n^2 + 2\,n + 3$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(4\,n)!\left(\frac{5\,n}{2} + \frac{3}{8}\right)}{n!^4\,(-1024)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{3\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{23}{20}\right],\left[\frac{3}{20},1,1\right],-\frac{1}{4}\right)}{8}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{10}}{6}+\frac{I\sqrt{2}}{6}$, primitive degree $=3$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{5}}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$2n^2+2n+3$$

SECONDS = .250

****

2, 2

SPECIAL VALUE of $z=\frac{1}{9}$

multiplier $=\frac{\sqrt{3}}{3}$, tau $=\frac{I}{2}\sqrt{6}$, degree $=3$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$2n^2+3$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(4n)!\left(\frac{4\sqrt{3}\,n}{3}+\frac{\sqrt{3}}{6}\right)}{n!^4\,2304^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2},\frac{3}{4},\frac{9}{8}\right],\left[\frac{1}{8},1,1\right],\frac{1}{9}\right)}{6}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{3}}{3}$, primitive degree $=3$, primitive tau $=\frac{I}{2}\sqrt{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$2n^2+3$$

SECONDS = .219

** **(17)**

```
> RamaPi(3, 2, 1, RUSSELL);
  RamaPi(3, 5, 1, RUSSELL);
  RamaPi(3, 7, 1, RUSSELL);
  RamaPi(3, 11, 1, RUSSELL);
  RamaPi(3, 13, 1, RUSSELL);
  RamaPi(3, 23, 1, RUSSELL);
```

RUSSELL

LEVEL $=3$, DEGREE $=2$

MODULAR EQUATION

$u^3=\alpha\beta$, $v^3=(1-\alpha)(1-\beta)$, $P(u,v)=0$,

$P(u,v)=u+v-1$

SECONDS = .687

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -4$$

$$z_0 = \frac{1}{2}$$

SECONDS = .63e-1

1, 2

SPECIAL VALUE of $z = \frac{1}{2}$

multiplier $= \frac{\sqrt{2}}{2}$, tau $= \frac{I}{3}\sqrt{6}$, degree $= 2$

$$\delta = 1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$3n^2 + 2$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3n)!\,(2n)!\left(\frac{2\sqrt{3}\,n}{3} + \frac{\sqrt{3}}{9}\right)}{n!^5\,216^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{7}{6}\right], \left[\frac{1}{6}, 1, 1\right], \frac{1}{2}\right)}{9} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{2}}{2}$, primitive degree $= 2$, primitive tau $= \frac{I}{3}\sqrt{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$3n^2 + 2$$

SECONDS = .375

****

2, 2

SPECIAL VALUE of $z = -4$

multiplier $= \frac{\sqrt{5}}{4} - \frac{I\sqrt{3}}{4}$, tau $= \frac{I\sqrt{15}}{6} + \frac{1}{2}$, degree $= 2$

$$\delta = 1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$3n^2 + 3n + 2$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3n)!\,(2n)!\left(\frac{5\sqrt{3}\,n}{3} + \frac{4\sqrt{3}}{9}\right)}{n!^5\,(-27)^n} = \infty$$

HYPERGEOMETRIC FORM

$$\frac{4\sqrt{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{19}{15}\right],\left[\frac{4}{15},1,1\right],-4\right)}{9}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{5}}{4}+\frac{I\sqrt{3}}{4}$, primitive degree $=2$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{5}\sqrt{3}}{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$3n^2+3n+2$$

SECONDS = .484

****

RUSSELL

LEVEL $=3$, DEGREE $=5$

MODULAR EQUATION

$$u^6=\alpha\beta,\quad v^6=(1-\alpha)(1-\beta),\quad P(u,v)=0,$$
$$P(u,v)=u^2+3uv+v^2-1$$

SECONDS = 1.250

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0=-4$$
$$z_0=-\frac{1}{16}$$
$$z_0=\frac{1}{2}$$
$$z_0=\frac{4}{125}$$

SECONDS = .281

1, 4

SPECIAL VALUE of $z=-4$

multiplier $=\frac{\sqrt{5}}{5}$, tau $=\frac{I}{6}\sqrt{15}$, degree $=5$

$$\delta=\frac{1}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$12n^2+5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(3n)!\,(2n)!\left(\frac{5\sqrt{3}\,n}{3}+\frac{4\sqrt{3}}{9}\right)}{n!^5\,(-27)^n}=\infty$$

HYPERGEOMETRIC FORM

$$\frac{4\sqrt{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{19}{15}\right],\left[\frac{4}{15},1,1\right],-4\right)}{9}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{5}}{4}+\frac{I\sqrt{3}}{4}$, primitive degree $=2$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{5}\sqrt{3}}{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$3n^2+3n+2$$

SECONDS = .250

****

2, 4

SPECIAL VALUE of z $=\frac{4}{125}$

multiplier $=\frac{\sqrt{5}}{5}$, tau $=\frac{I}{3}\sqrt{15}$, degree $=5$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$3n^2+5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(3n)!\,(2n)!\left(\frac{22\sqrt{3}\,n}{15}+\frac{8\sqrt{3}}{45}\right)}{n!^5\,3375^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{8\sqrt{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{37}{33}\right],\left[\frac{4}{33},1,1\right],\frac{4}{125}\right)}{45}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{5}}{5}$, primitive degree $=5$, primitive tau $=\frac{I}{3}\sqrt{15}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$3n^2+5$$

SECONDS = .313

****

3, 4

SPECIAL VALUE of z $=\frac{1}{2}$

multiplier $=\frac{\sqrt{2}}{5}-\frac{I\sqrt{3}}{5}$, tau $=\frac{I\sqrt{6}}{3}+1$, degree $=5$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$3n^2+2$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(3n)!\,(2n)!\left(\frac{2\sqrt{3}\,n}{3}+\frac{\sqrt{3}}{9}\right)}{n!^5\,216^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{7}{6}\right],\left[\frac{1}{6},1,1\right],\frac{1}{2}\right)}{9}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{2}}{2}$, primitive degree $=2$, primitive tau $=\frac{I}{3}\sqrt{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$3n^2+2$$

SECONDS = .406

****

4, 4

SPECIAL VALUE of $z=-\frac{1}{16}$

multiplier $=\frac{\sqrt{17}}{10}-\frac{I\sqrt{3}}{10}$, tau $=\frac{I\sqrt{51}}{6}+\frac{1}{2}$, degree $=5$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$3n^2+3n+5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(3n)!\,(2n)!\left(\frac{17\sqrt{3}\,n}{12}+\frac{7\sqrt{3}}{36}\right)}{n!^5\,(-1728)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{7\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{58}{51}\right],\left[\frac{7}{51},1,1\right],-\frac{1}{16}\right)}{36}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{17}}{10}+\frac{I\sqrt{3}}{10}$, primitive degree $=5$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{17}\sqrt{3}}{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$3n^2+3n+5$$

SECONDS = .703

****

RUSSELL

LEVEL $=3$, DEGREE $=7$

MODULAR EQUATION

$$u^2=\alpha\beta,\quad v^2=(1-\alpha)(1-\beta),\quad P(u,v)=0,$$

$$P(u,v)=-u^8+328u^7v-35644u^6v^2+1259384u^5v^3+2590714u^4v^4+1259384u^3v^5-35644u^2v^6+328uv^7-v^8+4u^6+8736u^5v+452316u^4v^2+477568u^3v^3+452316u^2v^4+8736uv^5+4v^6-6u^4+9975u^3v-79878u^2v^2+9975uv^3-6v^4+4u^2+644uv+4v^2-1$$

SECONDS = 19.422

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{1}{80}$$

$$z_0 = \frac{2}{27}$$

SECONDS = .234

1, 2

SPECIAL VALUE of $z = -\frac{1}{80}$

multiplier $= \frac{5}{14} + \frac{I\sqrt{3}}{14}$, tau $= -\frac{1}{2} + \frac{5\,I\sqrt{3}}{6}$, degree $= 7$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$3\,n^2 + 3\,n + 7$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3\,n)!\,(2\,n)!\left(\frac{3\sqrt{15}\,n}{4} + \frac{\sqrt{15}}{12}\right)}{n!^5\,(-8640)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{15}\ \mathrm{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{10}{9}\right], \left[\frac{1}{9}, 1, 1\right], -\frac{1}{80}\right)}{12} = \frac{1}{\pi}$$

primitive multiplier $= \frac{5}{14} + \frac{I\sqrt{3}}{14}$, primitive degree $= 7$, primitive tau $= -\frac{1}{2} + \frac{5\,I\sqrt{3}}{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$3\,n^2 + 3\,n + 7$

SECONDS = 2.469

****

2, 2

SPECIAL VALUE of $z = \frac{2}{27}$

multiplier $= \frac{2}{7} - \frac{I\sqrt{3}}{7}$, tau $= \frac{2\,I\sqrt{3}}{3} + 1$, degree $= 7$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$3\,n^2 + 4$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3\,n)!\,(2\,n)!\left(\frac{20\,n}{9} + \frac{8}{27}\right)}{n!^5\,1458^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{8\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{17}{15}\right],\left[\frac{2}{15},1,1\right],\frac{2}{27}\right)}{27}=\frac{1}{\pi}$$

primitive multiplier $=\frac{1}{2}$, primitive degree $=4$, primitive tau $=\frac{2\,I}{3}\sqrt{3}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta=1$

$$3\,n^2+4$$

SECONDS = 8.907

****

RUSSELL

LEVEL $=3$, DEGREE $=11$

MODULAR EQUATION

$$u^6=\alpha\,\beta,\quad v^6=(1-\alpha)\,(1-\beta),\quad P(u,v)=0,$$
$$P(u,v)=-u^4+15\,u^3\,v+16\,u^2\,v^2+15\,u\,v^3-v^4+2\,u^2+12\,u\,v+2\,v^2-1$$

SECONDS = 3.156

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0=-\frac{1}{16}$$
$$z_0=-\frac{1}{1024}$$
$$z_0=\frac{1}{2}$$

SECONDS = .563

1, 3

SPECIAL VALUE of z $=-\frac{1}{16}$

multiplier $=\frac{\sqrt{17}}{22}+\frac{3\,I\sqrt{3}}{22}$, tau $=\frac{I\sqrt{51}}{6}-\frac{3}{2}$, degree $=11$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$3\,n^2+3\,n+5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(3\,n)!\,(2\,n)!\left(\frac{17\sqrt{3}\,n}{12}+\frac{7\sqrt{3}}{36}\right)}{n!^5\,(-1728)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{7\sqrt{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{58}{51}\right],\left[\frac{7}{51},1,1\right],-\frac{1}{16}\right)}{36}=\frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{17}}{10} + \frac{I\sqrt{3}}{10}$, primitive degree $= 5$, primitive tau $= -\frac{1}{2} + \frac{I\sqrt{17}\sqrt{3}}{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$3n^2 + 3n + 5$$

SECONDS = 1.235

****

2, 3

SPECIAL VALUE of $z = \frac{1}{2}$

multiplier $= \frac{2\sqrt{2}}{11} + \frac{I\sqrt{3}}{11}$, tau $= \frac{I\sqrt{6}}{3} - \frac{1}{2}$, degree $= 11$

$$\delta = \frac{1}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$12n^2 + 12n + 11$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3n)!\,(2n)!\left(\frac{2\sqrt{3}\,n}{3} + \frac{\sqrt{3}}{9}\right)}{n!^5\, 216^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\ \text{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{7}{6}\right], \left[\frac{1}{6}, 1, 1\right], \frac{1}{2}\right)}{9} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{2}}{2}$, primitive degree $= 2$, primitive tau $= \frac{I}{3}\sqrt{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$3n^2 + 2$$

SECONDS = .437

****

3, 3

SPECIAL VALUE of $z = -\frac{1}{1024}$

multiplier $= \frac{\sqrt{41}}{22} - \frac{I\sqrt{3}}{22}$, tau $= \frac{I\sqrt{123}}{6} + \frac{1}{2}$, degree $= 11$

$$\delta = 1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$3n^2 + 3n + 11$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3n)!\,(2n)!\left(\frac{205\sqrt{3}\,n}{96} + \frac{53\sqrt{3}}{288}\right)}{n!^5\,(-110592)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{53\sqrt{3}\,\mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{668}{615}\right],\left[\frac{53}{615},1,1\right],-\frac{1}{1024}\right)}{288}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{41}}{22}+\frac{I\sqrt{3}}{22}$, primitive degree $=11$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{3}\sqrt{41}}{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$3n^2+3n+11$$

SECONDS = .578

****

RUSSELL

LEVEL $=3$, DEGREE $=13$

MODULAR EQUATION

$$u^2=\alpha\beta,\quad v^2=(1-\alpha)(1-\beta),\quad P(u,v)=0,$$

$$\begin{aligned}P(u,v)={}&u^{14}+76142u^{13}v+1932468187u^{12}v^2+16346295812652u^{11}v^3-42859027901079u^{10}v^4+30681672585330u^9v^5\\&+44443969755835u^8v^6-90882188302360u^7v^7+44443969755835u^6v^8+30681672585330u^5v^9-42859027901079u^4v^{10}\\&+16346295812652u^3v^{11}+1932468187u^2v^{12}+76142uv^{13}+v^{14}-7u^{12}+11747580u^{11}v-189423188910u^{10}v^2\\&-22154358399732u^9v^3+165532621704183u^8v^4-118239427045896u^7v^5-206947164829828u^6v^6-118239427045896u^5v^7\\&+165532621704183u^4v^8-22154358399732u^3v^9-189423188910u^2v^{10}+11747580uv^{11}-7v^{12}+21u^{10}+106764996u^9v\\&+542495769825u^8v^2+5640646277136u^7v^3-70407752237430u^6v^4+190989688889688u^5v^5-70407752237430u^4v^6\\&+5640646277136u^3v^7+542495769825u^2v^8+106764996uv^9+21v^{10}-35u^8+187172492u^7v-331040877068u^6v^2\\&+2589434377140u^5v^3-11347484743458u^4v^4+2589434377140u^3v^5-331040877068u^2v^6+187172492uv^7-35v^8+35u^6\\&+75901566u^5v+32147569134u^4v^2+163984049190u^3v^3+32147569134u^2v^4+75901566uv^5+35v^6-21u^4+5732844u^3v\\&-151003875u^2v^2+5732844uv^3-21v^4+7u^2+24869uv+7v^2-1\end{aligned}$$

SECONDS = 13342.187

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0=-\frac{1}{80}$$

$$z_0=-\frac{1}{3024}$$

SECONDS = .500

1, 2

SPECIAL VALUE of $z=-\frac{1}{80}$

multiplier $=\frac{3I\sqrt{3}}{26}+\frac{5}{26}$, tau $=-\frac{3}{2}+\frac{5I\sqrt{3}}{6}$, degree $=13$

$$\delta=1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$3n^2+3n+7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3n)!\,(2n)!\left(\frac{3\sqrt{15}\,n}{4}+\frac{\sqrt{15}}{12}\right)}{n!^5\,(-8640)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{15}\ \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{10}{9}\right],\left[\frac{1}{9},1,1\right],-\frac{1}{80}\right)}{12}=\frac{1}{\pi}$$

primitive multiplier $=\frac{5}{14}+\frac{I\sqrt{3}}{14}$, primitive degree $=7$, primitive tau $=-\frac{1}{2}+\frac{5\,I\sqrt{3}}{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$3n^2+3n+7$$

SECONDS = 4.375

****

2, 2

SPECIAL VALUE of z $=-\frac{1}{3024}$

multiplier $=\frac{7}{26}+\frac{I\sqrt{3}}{26}$, tau $=-\frac{1}{2}+\frac{7\,I\sqrt{3}}{6}$, degree $=13$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$3n^2+3n+13$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3n)!\,(2n)!\left(\frac{55\sqrt{7}\,n}{36}+\frac{13\sqrt{7}}{108}\right)}{n!^5\,(-326592)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{13\sqrt{7}\ \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{178}{165}\right],\left[\frac{13}{165},1,1\right],-\frac{1}{3024}\right)}{108}=\frac{1}{\pi}$$

primitive multiplier $=\frac{7}{26}+\frac{I\sqrt{3}}{26}$, primitive degree $=13$, primitive tau $=-\frac{1}{2}+\frac{7\,I\sqrt{3}}{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$3n^2+3n+13$$

SECONDS = 4.407

****

RUSSELL

LEVEL $=3$, DEGREE $=23$

MODULAR EQUATION

$$u^6=\alpha\beta,\quad v^6=(1-\alpha)(1-\beta),\quad P(u,v)=0,$$

$$P(u,v)=-u^8+606u^7v-1201u^6v^2-450u^5v^3+2460u^4v^4-450u^3v^5-1201u^2v^6+606uv^7-v^8+4u^6+828u^5v-348u^4v^2$$
$$-936u^3v^3-348u^2v^4+828uv^5+4v^6-6u^4+657u^3v-642u^2v^2+657uv^3-6v^4+4u^2+96uv+4v^2-1$$

SECONDS = 27.437

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -4$$

$$z_0 = -\frac{1}{16}$$

$$z_0 = -\frac{1}{250000}$$

$$z_0 = \frac{4}{125}$$

SECONDS = 1.312

1, 4

SPECIAL VALUE of $z = \frac{4}{125}$

$$\text{multiplier} = \frac{2\sqrt{5}}{23} + \frac{I\sqrt{3}}{23}, \quad \text{tau} = \frac{I\sqrt{15}}{3} - \frac{1}{2}, \quad \text{degree} = 23$$

$$\delta = \frac{1}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$12\,n^2 + 12\,n + 23$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3\,n)!\,(2\,n)!\left(\frac{22\sqrt{3}\,n}{15} + \frac{8\sqrt{3}}{45}\right)}{n!^5\,3375^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{8\sqrt{3}\,\text{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{37}{33}\right], \left[\frac{4}{33}, 1, 1\right], \frac{4}{125}\right)}{45} = \frac{1}{\pi}$$

$$\text{primitive multiplier} = \frac{\sqrt{5}}{5}, \quad \text{primitive degree} = 5, \quad \text{primitive tau} = \frac{I}{3}\sqrt{15}$$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$3\,n^2 + 5$$

SECONDS = .547

****

2, 4

SPECIAL VALUE of $z = -4$

$$\text{multiplier} = \frac{2\sqrt{5}}{23} + \frac{I\sqrt{3}}{23}, \quad \text{tau} = \frac{I\sqrt{15}}{6} - \frac{1}{4}, \quad \text{degree} = 23$$

$$\delta = \frac{1}{4}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$48\,n^2 + 24\,n + 23$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3\,n)!\,(2\,n)!\left(\frac{5\sqrt{3}\,n}{3}+\frac{4\sqrt{3}}{9}\right)}{n!^5\,(-27)^n}=\infty$$

HYPERGEOMETRIC FORM

$$\frac{4\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{19}{15}\right],\left[\frac{4}{15},1,1\right],-4\right)}{9}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{5}}{4}+\frac{I\sqrt{3}}{4}$, primitive degree $=2$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{5}\,\sqrt{3}}{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$3\,n^2+3\,n+2$

SECONDS = .375

****

3, 4

SPECIAL VALUE of $z=-\frac{1}{16}$

multiplier $=\frac{\sqrt{17}}{46}+\frac{5\,I\sqrt{3}}{46}$, tau $=\frac{I\sqrt{51}}{6}-\frac{5}{2}$, degree $=23$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$3\,n^2+3\,n+5$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3\,n)!\,(2\,n)!\left(\frac{17\sqrt{3}\,n}{12}+\frac{7\sqrt{3}}{36}\right)}{n!^5\,(-1728)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{7\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{58}{51}\right],\left[\frac{7}{51},1,1\right],-\frac{1}{16}\right)}{36}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{17}}{10}+\frac{I\sqrt{3}}{10}$, primitive degree $=5$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{17}\,\sqrt{3}}{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$3\,n^2+3\,n+5$

SECONDS = .438

****

4, 4

SPECIAL VALUE of $z=-\frac{1}{250000}$

multiplier $=\frac{\sqrt{89}}{46}-\frac{I\sqrt{3}}{46}$, tau $=\frac{I\sqrt{267}}{6}+\frac{1}{2}$, degree $=23$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$3\,n^2+3\,n+23$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3\,n)!\,(2\,n)!\left(\frac{4717\sqrt{3}\,n}{1500}+\frac{827\sqrt{3}}{4500}\right)}{n!^5\,(-27000000)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{827\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{3},\frac{1}{2},\frac{2}{3},\frac{14978}{14151}\right],\left[\frac{827}{14151},1,1\right],-\frac{1}{250000}\right)}{4500}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{89}}{46}+\frac{I\sqrt{3}}{46}$, primitive degree $=23$, primitive tau $=-\frac{1}{2}+\frac{I\sqrt{3}\sqrt{89}}{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$3\,n^2+3\,n+23$$

SECONDS = .594

** (18)

```
> RamaPi(4, 5, 1, RUSSELL);
  RamaPi(4, 5, 1, WEBER);
```

RUSSELL

LEVEL $=4$, DEGREE $=5$

MODULAR EQUATION

$u^2=\alpha\,\beta$, $v^2=(1-\alpha)\,(1-\beta)$, *P(u,v)=0,*

$$P(u,v)=u^3+3\,u^2\,v+3\,u\,v^2+v^3-3\,u^2+26\,u\,v-3\,v^2+3\,u+3\,v-1$$

SECONDS = 1.484

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0=-\frac{1}{8}$$

SECONDS = .15e-1

1, 1

SPECIAL VALUE of z $=-\frac{1}{8}$

multiplier $=\frac{2}{5}-\frac{I}{5}$, tau $=\frac{1}{2}+I$, degree $=5$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$4\,n^2+4\,n+5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(2n)!^3 \left(\frac{3\sqrt{2}\,n}{2} + \frac{\sqrt{2}}{4}\right)}{n!^6 \, (-512)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{2}\ \text{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{7}{6}\right], \left[\frac{1}{6}, 1, 1\right], -\frac{1}{8}\right)}{4} = \frac{1}{\pi}$$

primitive multiplier $= \frac{2}{5} + \frac{I}{5}$, primitive degree $= 5$, primitive tau $= -\frac{1}{2} + I$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$$4\,n^2 + 4\,n + 5$$

SECONDS = .204

****

WEBER

LEVEL $= 4$, DEGREE $= 5$

MODULAR EQUATION

$$\alpha\,(1-\alpha) = \frac{16}{u^{12}},\ \beta\,(1-\beta) = \frac{16}{v^{12}},\ P(u,v)=0,$$

$$P(u,v) = u\,v\,(u^2\,v^2 - 4)^2 - (u^3 + v^3)^2$$

SECONDS = .31e-1

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -8$$

$$z_0 = -\frac{1}{8}$$

SECONDS = .141

1, 2

SPECIAL VALUE of $z = -8$

multiplier $= \frac{2}{5} - \frac{I}{5}$, tau $= \frac{1}{4} + \frac{I}{2}$, degree $= 5$

$$\delta = \frac{1}{2}$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$16\,n^2 + 8\,n + 5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(2n)!^3\,(3n+1)}{n!^6\,(-8)^n} = \infty$$

HYPERGEOMETRIC FORM

$$\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{4}{3}\right],\left[\frac{1}{3},1,1\right],-8\right)=\frac{1}{\pi}$$

primitive multiplier $=\frac{1}{2}+\frac{I}{2}$, primitive degree $=2$, primitive tau $=-\frac{1}{2}+\frac{I}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$4n^2+4n+2$$

SECONDS = .172

****

2, 2

SPECIAL VALUE of $z=-\frac{1}{8}$

multiplier $=\frac{2}{5}+\frac{I}{5}$, tau $=-\frac{1}{2}+I$, degree $=5$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$4n^2+4n+5$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(2n)!^3\left(\frac{3\sqrt{2}\,n}{2}+\frac{\sqrt{2}}{4}\right)}{n!^6\,(-512)^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{2}\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{7}{6}\right],\left[\frac{1}{6},1,1\right],-\frac{1}{8}\right)}{4}=\frac{1}{\pi}$$

primitive multiplier $=\frac{2}{5}+\frac{I}{5}$, primitive degree $=5$, primitive tau $=-\frac{1}{2}+I$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$4n^2+4n+5$$

SECONDS = .203

** **(19)**

```
> RamaPi(4, 7, 1, WEBER);
  RamaPi(4, 7, 1, RUSSELL);
```

WEBER

LEVEL $=4$, DEGREE $=7$

MODULAR EQUATION

$$\alpha(1-\alpha)=\frac{16}{u^{12}},\ \beta(1-\beta)=\frac{16}{v^{12}},\ P(u,v)=0,$$

$$P(u,v)=u\,v\,(u^3v^3+8)^2-(u^4+7u^2v^2+v^4)^2$$

SECONDS = .31e-1

SPECIAL (OR SINGULAR) VALUES OF z

$z_0 = 4$

$z_0 = 64$

$z_0 = \frac{1}{4}$

$z_0 = \frac{1}{64}$

SECONDS = .282

1, 4

SPECIAL VALUE of $z = \frac{1}{64}$

multiplier $= \frac{\sqrt{7}}{7}$, tau $= \frac{I}{2}\sqrt{7}$, degree $= 7$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$4n^2 + 7$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(2n)!^3 \left(\frac{21n}{8} + \frac{5}{16}\right)}{n!^6\, 4096^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{5\,\mathrm{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{47}{42}\right], \left[\frac{5}{42}, 1, 1\right], \frac{1}{64}\right)}{16} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{7}}{7}$, primitive degree $= 7$, primitive tau $= \frac{I}{2}\sqrt{7}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$4n^2 + 7$

SECONDS = .250

****

2, 4

SPECIAL VALUE of $z = 64$

multiplier $= \frac{\sqrt{7}}{7}$, tau $= \frac{I}{8}\sqrt{7}$, degree $= 7$

$\delta = \frac{1}{4}$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$64n^2 + 7$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(2n)!^3 \left(\frac{21 I n}{4} + 2 I\right)}{n!^6} = \infty$$

HYPERGEOMETRIC FORM

$$2 I \operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{29}{21}\right], \left[\frac{8}{21}, 1, 1\right], 64\right) = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{7}}{4} + \frac{3 I}{4}$, primitive degree $= 1$, primitive tau $= \frac{I\sqrt{7}}{8} - \frac{3}{8}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$$4 n^2 + 3 n + 1$$

SECONDS = .188

****

3, 4

SPECIAL VALUE of $z = \frac{1}{4}$

multiplier $= \frac{\sqrt{3}}{7} - \frac{2 I}{7}$, tau $= \frac{I\sqrt{3}}{2} + 1$, degree $= 7$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$4 n^2 + 3$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(2n)!^3 \left(\frac{3n}{2} + \frac{1}{4}\right)}{n!^6\, 256^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\operatorname{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{7}{6}\right], \left[\frac{1}{6}, 1, 1\right], \frac{1}{4}\right)}{4} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{3}}{3}$, primitive degree $= 3$, primitive tau $= \frac{I}{2}\sqrt{3}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$$4 n^2 + 3$$

SECONDS = .234

****

4, 4

SPECIAL VALUE of $z = 4$

multiplier $= \frac{\sqrt{3}}{7} - \frac{2 I}{7}$, tau $= \frac{I\sqrt{3}}{4} + \frac{1}{2}$, degree $= 7$

$\delta = \frac{1}{2}$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$16 n^2 + 16 n + 7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(2n)!^3 \left(\frac{3 I n}{2} + \frac{I}{2}\right)}{n!^6\, 16^n} = \infty$$

HYPERGEOMETRIC FORM

$$\frac{I}{2} \text{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{4}{3}\right], \left[\frac{1}{3}, 1, 1\right], 4\right) = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{3}}{2} + \frac{I}{2}$, primitive degree $= 1$, primitive tau $= -\frac{1}{4} + \frac{I\sqrt{3}}{4}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$$4n^2 + 2n + 1$$

SECONDS = .203

****

RUSSELL

LEVEL $= 4$, DEGREE $= 7$

MODULAR EQUATION

$$u^8 = \alpha\beta, \quad v^8 = (1-\alpha)(1-\beta), \quad P(u,v)=0,$$

$$P(u,v) = u + v - 1$$

SECONDS = .656

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = \frac{1}{4}$$

$$z_0 = \frac{1}{64}$$

SECONDS = .16e-1

1, 2

SPECIAL VALUE of z $= \frac{1}{64}$

multiplier $= \frac{\sqrt{7}}{7}$, tau $= \frac{I}{2}\sqrt{7}$, degree $= 7$

$$\delta = 1$$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$4n^2 + 7$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(2n)!^3 \left(\frac{21 n}{8} + \frac{5}{16}\right)}{n!^6\, 4096^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{5\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{47}{42}\right],\left[\frac{5}{42},1,1\right],\frac{1}{64}\right)}{16}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{7}}{7}$, primitive degree $=7$, primitive tau $=\frac{I}{2}\sqrt{7}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$4n^2+7$$

SECONDS = .234

****

2, 2

SPECIAL VALUE of z $=\frac{1}{4}$

multiplier $=\frac{\sqrt{3}}{7}-\frac{2I}{7}$, tau $=\frac{I\sqrt{3}}{2}+1$, degree $=7$

$\delta=1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$4n^2+3$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty}\frac{(2n)!^3\left(\frac{3n}{2}+\frac{1}{4}\right)}{n!^6\,256^n}=\frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{1}{2},\frac{1}{2},\frac{7}{6}\right],\left[\frac{1}{6},1,1\right],\frac{1}{4}\right)}{4}=\frac{1}{\pi}$$

primitive multiplier $=\frac{\sqrt{3}}{3}$, primitive degree $=3$, primitive tau $=\frac{I}{2}\sqrt{3}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta=1$

$$4n^2+3$$

SECONDS = .187

** **(20)**

```
> RamaPi(4, 3, 1, RUSSELL);
```

RUSSELL

LEVEL $=4$, DEGREE $=3$

MODULAR EQUATION

$$u^4=\alpha\beta,\quad v^4=(1-\alpha)(1-\beta),\quad P(u,v)=0,$$
$$P(u,v)=u+v-1$$

SECONDS = .719

SPECIAL (OR SINGULAR) VALUES OF z

$z_0 = -1$

$z_0 = \frac{1}{4}$

SECONDS = 0.

1, 2

SPECIAL VALUE of $z = \frac{1}{4}$

multiplier $= \frac{\sqrt{3}}{3}$, tau $= \frac{I}{2}\sqrt{3}$, degree $= 3$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$4n^2 + 3$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(2n)!^3 \left(\frac{3n}{2} + \frac{1}{4}\right)}{n!^6\, 256^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\text{hypergeom}\left(\left[\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, \frac{7}{6}\right], \left[\frac{1}{6}, 1, 1\right], \frac{1}{4}\right)}{4} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{3}}{3}$, primitive degree $= 3$, primitive tau $= \frac{I}{2}\sqrt{3}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$4n^2 + 3$

SECONDS = .297

****

2, 2

SPECIAL VALUE of $z = -1$

multiplier $= \frac{\sqrt{2}}{3} - \frac{I}{3}$, tau $= \frac{I\sqrt{2}}{2} + \frac{1}{2}$, degree $= 3$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$4n^2 + 4n + 3$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(2n)!^3 \left(2n + \frac{1}{2}\right)}{n!^6\, (-64)^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{1}{\pi} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{2}}{3} + \frac{I}{3}$, primitive degree $= 3$, primitive tau $= \frac{I\sqrt{2}}{2} - \frac{1}{2}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$$4\, n^2 + 4\, n + 3$$

SECONDS = .188

(21)

```
> RamaPi(3,2,1,RUSSELL);
```

RUSSELL

LEVEL = 3, DEGREE = 2

MODULAR EQUATION

$$u^3 = \alpha\,\beta,\quad v^3 = (1-\alpha)\,(1-\beta),\quad P(u,v)=0,$$
$$P(u,v) = u + v - 1$$

SECONDS = .641

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -4$$
$$z_0 = \frac{1}{2}$$

SECONDS = .15e-1

1, 2

SPECIAL VALUE of z $= \frac{1}{2}$

multiplier $= \frac{\sqrt{2}}{2}$, tau $= \frac{I}{3}\sqrt{6}$, degree $= 2$

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\ \delta$

$$3\, n^2 + 2$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3\,n)!\,(2\,n)!\left(\frac{2\sqrt{3}\,n}{3} + \frac{\sqrt{3}}{9}\right)}{n!^5\, 216^n} = \frac{1}{\pi}$$

HYPERGEOMETRIC FORM

$$\frac{\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{7}{6}\right], \left[\frac{1}{6}, 1, 1\right], \frac{1}{2}\right)}{9} = \frac{1}{\pi}$$

primitive multiplier $= \frac{\sqrt{2}}{2}$, primitive degree $= 2$, primitive tau $= \frac{I}{3}\sqrt{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\ \delta = 1$

$$3\, n^2 + 2$$

SECONDS = .250

****

2, 2

SPECIAL VALUE of z = -4

multiplier = $\frac{\sqrt{5}}{4} - \frac{I\sqrt{3}}{4}$, tau = $\frac{I\sqrt{15}}{6} + \frac{1}{2}$, degree = 2

$\delta = 1$

sequence of possible degrees for n=0,1,2,... corresponding to the same $+\delta$

$$3n^2 + 3n + 2$$

RAMANUJAN SERIES

$$\sum_{n=0}^{\infty} \frac{(3n)!\,(2n)!\left(\frac{5\sqrt{3}\,n}{3} + \frac{4\sqrt{3}}{9}\right)}{n!^5\,(-27)^n} = \infty$$

HYPERGEOMETRIC FORM

$$\frac{4\sqrt{3}\ \mathrm{hypergeom}\left(\left[\frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{19}{15}\right], \left[\frac{4}{15}, 1, 1\right], -4\right)}{9} = \frac{1}{\pi}$$

primitive multiplier = $\frac{\sqrt{5}}{4} + \frac{I\sqrt{3}}{4}$, primitive degree = 2, primitive tau = $-\frac{1}{2} + \frac{I\sqrt{5}\sqrt{3}}{6}$

sequence of possible possible degrees for n=0,1,2,... corresponding to $+\delta = 1$

$$3n^2 + 3n + 2$$

SECONDS = .359

*** (22)

```
> RamaPi(4,3,1,WEBER);
```

WEBER

LEVEL = 4, DEGREE = 3

MODULAR EQUATION

$$\alpha(1-\alpha) = \frac{16}{u^{12}}, \beta(1-\beta) = \frac{16}{v^{12}}, P(u,v)=0,$$

$$P(u,v) = -227859025\, v^4 + 319908996\, u\, v$$

SECONDS = .16e-1

SPECIAL (OR SINGULAR) VALUES OF z

$$z_0 = -\frac{16748608962367633368803451796651201231236148736064}{139957897043682545998498068096198498278847900390625}$$

$$z_0 = \frac{16748608962367633368803451796651201231236148736064}{139957897043682545998498068096198498278847900390625}$$

SECONDS = .719

```
Error, (in mba) numeric exception: division by zero
```