MY FORMULAS FOR $1/\pi^2$

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Identities 1 and 2

Let

$$A_n = \frac{\left(\frac{1}{2}\right)_n^5}{n!^5},$$

then

$$\sum_{n=0}^{\infty} (-1)^n A_n \frac{20n^2+8n+1}{2^{2n}} = \frac{8}{\pi^2}, \quad (1)$$

and

$$\sum_{n=0}^{\infty} (-1)^n A_n \frac{820n^2+180n+13}{2^{10n}} = \frac{128}{\pi^2}, \quad (2)$$

this identities have been proved by me.

Identity 3

Let

$$A_n = \frac{\left(\frac{1}{2}\right)_n^3 \left(\frac{1}{4}\right)_n \left(\frac{3}{4}\right)_n}{n!^5},$$

then

$$\sum_{n=0}^{\infty} A_n \frac{120n^2+34n+3}{2^{4n}} = \frac{32}{\pi^2}. \tag{3}$$

this identity has been proved by me.

Identity 4

Let

$$A_n = \frac{\left(\frac{1}{2}\right)_n \left(\frac{1}{4}\right)_n \left(\frac{3}{4}\right)_n \left(\frac{1}{3}\right)_n \left(\frac{2}{3}\right)_n}{n!^5},$$

then

$$\sum_{n=0}^{\infty} (-1)^n A_n \frac{252n^2 + 63n + 5}{48^n} = \frac{48}{\pi^2}, \qquad (4)$$

this identity remain unproved.

Identity 5

Let

$$A_n = \frac{\left(\frac{1}{2}\right)_n \left(\frac{1}{4}\right)_n \left(\frac{3}{4}\right)_n \left(\frac{1}{6}\right)_n \left(\frac{5}{6}\right)_n}{n!^5},$$

then

$$\sum_{n=0}^{\infty} (-1)^n A_n \frac{1640n^2 + 278n + 15}{2^{10n}} = \frac{256\sqrt{3}}{3\pi^2}, \qquad (5)$$

this identity remain unproved.

Identity 6

Let

$$A_n = \frac{\left(\frac{1}{2}\right)_n \left(\frac{1}{8}\right)_n \left(\frac{3}{8}\right)_n \left(\frac{5}{8}\right)_n \left(\frac{7}{8}\right)_n}{n!^5},$$

then

$$\sum_{n=0}^{\infty} A_n \frac{1920n^2 + 304n + 15}{7^{4n}} = \frac{56\sqrt{7}}{\pi^2}, \tag{6}$$

this identity remain unproved.

Identity 7

Let

$$A_n = \frac{\left(\frac{1}{2}\right)_n \left(\frac{1}{3}\right)_n \left(\frac{2}{3}\right)_n \left(\frac{1}{6}\right)_n \left(\frac{5}{6}\right)_n}{n!^5},$$

then

$$\sum_{n=0}^{\infty} (-1)^n A_n \frac{5418n^2 + 693n + 29}{80^{3n}} = \frac{128\sqrt{5}}{\pi^2}, \qquad (7)$$

this identity remain unproved.

Identity 8

Let

$$A_n = \binom{2n}{n}^2 \sum_{j=0}^{n} \binom{2n-2j}{n-j}^2 \binom{2j}{j}^2,$$

then

$$\sum_{n=0}^{\infty} A_n \frac{36n^2+12n+1}{2^{10n}} = \frac{32}{\pi^2}, \tag{8}$$

this identity remain unproved.